GCSE MATHEMATICS

Aiming for Grade 5

REVISION BOOKLET

Exam Dates:

_____

_____

_____

Name: _____

Teacher: _____

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Laws of Indices

Things to remember:

- The exam question will use the word "simplify".
- If the exam question has the words "work out the value of", or "evaluate" it means the answer is a number.

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^0 = 1$$

$$a^{-m} = \frac{1}{a^m}$$

Questions:

1. Evaluate:

   a) $2^3 \times 5^2$

   ....200.......................
   (1)

   b) $4^5 \div 4^3$

   ....16.......................
   (1)

   c) 8^0

   ....1.......................
   (1)

   d) 2^{-3}

   ....$\frac{1}{8}$.......................
   (1)
   (Total 4 marks)

2. Simplify:

   a) $x^3 \times x^4$

   ....x^7.......................
   (1)

   b) $(y^3)^2$

   ....y^6.......................
   (1)

   c) $\dfrac{m^7}{m^5}$

   ....m^2.......................
   (1)
   (Total 3 marks)

3. Simplify:

   e) $\dfrac{p^4 \times p^2}{p^5}$

   ....p.......................
   (2)

   d) $3d^2 \times 4d^5$

   ....$12d^7$.......................
   (2)

   g) $(2k^4)^3$

   ....$8k^{12}$.......................
   (2)
   (Total 6 marks)

3

4. $64 = 4^x$
Work out the value of x

$x =$ ....**3**.............
(Total 1 mark)

5. $p^2 \times p^5 = p^x \times p^3$
Work out the value of x

$x =$ ....**4**.............
(Total 2 marks)

6. $\dfrac{m^3 \times m^x}{m^2} = m^6$
Work out the value of x

$x =$ ....**5**.............
(Total 2 marks)

7. $10^x = 1$
Work out the value of x

$x =$ ....**0**.............
(Total 1 mark)

8. Simplify:

a) $x^2 y^3 \times x y^4$

..............$x^3 y^7$..............
(2)

b) $3a^3 b^7 \times 2a^2 b^{-4}$

..............$6a^5 b^3$..............
(2)

c) $\dfrac{15m^6}{3m^3}$

..............$5m^3$..............
(2)
(Total 6 marks)

9. Evaluate:

a) x^0

..............**1**..............
(1)

b) $(6x)^0$

..............**1**..............
(1)

c) $6x^0$

..............**6**..............
(1)
(Total 3 marks)

Contents ⌂

10. Write these numbers in order of size.
Start with the smallest number.

5^2 2^5 5^0 2^1 $(2^2)^2$

25   32   1   2   16

$5^0, 2^1, (2^2)^2, 5^2, 2^5$

(Total 2 marks)

11. Express the following as a power of 2:

a)   8

2^3

(1)

b) 4^2 $(2^2)^2$

2^4

(2)

c) 4×8^2 $2^2 \times (2^3)^2$

2^8

(2)

c) $\dfrac{1}{16}$ $\dfrac{1}{2^4}$

2^{-4}

(2)
(Total 7 marks)

12. Simplify:

a) $5c^2d \times 2d$

$10c^2d^2$

(2)

b) $x^2 \times (x^3)^5$

x^{17}

(2)

c) $\dfrac{20m^4n^6}{10m^2n^2}$

$2m^2n^4$

(2)
(Total 6 marks)

13. Write down the reciprocal of 6

$\dfrac{1}{6}$

(Total 1 mark)

14. $(y^4)^b = y^{12}$
Work out the value of b

$b =$ 3

(Total 2 marks)

Standard Form

Things to remember:

- Standard form gives us a more concise way of writing really big or really small numbers.
- It must be in the form $a \times 10^n$ where $1 \leq a < 10$ and n is an integer.
- For example:
  - $45\,000\,000 = 4.5 \times 10^7$
  - $0.000\,05 = 5 \times 10^{-5}$

Questions:

1. Write as an ordinary number:

   a) 5×10^6

   5 000 000
   (1)

   b) 2.74×10^3

   2 740
   (1)

   c) 1.7×10^{-2}

   0.017
   (1)

   d) 9.763×10^{-5}

   0.000 097 63
   (1)
   (Total 4 marks)

2. Write in standard form:

   a) 3000

   3×10^3
   (1)

   b) 540 000

   5.4×10^5
   (1)

   c) 0.000 067

   6.7×10^{-5}
   (1)

   d) 0.009 31

   9.31×10^{-3}
   (1)
   (Total 4 marks)

3. Work out $(1.2 \times 10^5) \times (8 \times 10^4)$
   Give your answer in standard form.

   $1.2 \times 8 \times 10^5 \times 10^4$

   9.6×10^9
   (Total 2 marks)

6

4. Work out $(3 \times 10^6) \times (1.9 \times 10^{-2})$
   Give your answer as an ordinary number.

   $3 \times 1.9 \times 10^6 \times 10^{-2}$

   5.7×10^4
   ....
   (Total 2 marks)

5. Work out $(6 \times 10^7) \div (1.2 \times 10^8)$
   Give your answer as an ordinary number.

   $\dfrac{6 \times 10^7}{1.2 \times 10^8}$

   5×10^{-1}
   ....
   (Total 2 marks)

6. Work out $\dfrac{0.004 \times 0.05}{0.002}$
   Give your answer in standard form.

   $\dfrac{4 \times 10^{-3} \times 5 \times 10^{-2}}{2 \times 10^{-3}} = 10 \times 10^{-2}$

   1×10^{-1}
   ....
   (Total 2 marks)

7. Write these numbers in order of size.
   Start with the smallest number.

   62×10^3 0.62×10^0 620×10^{-2} 6.2×10^2

   62000 0.62 6.2 6200

   $0.62 \times 10^0, \ 620 \times 10^{-2}, \ 6.2 \times 10^2, \ 62 \times 10^3$
   ....
   (Total 2 marks)

8. One electron has a mass of 9.1×10^{-31} grams.
   Calculate the mass of 100 electrons. $9.1 \times 10^{-31} \times 10^2$
   Give your answer in standard form.

   9.1×10^{-29}
   ....
   (Total 2 marks)

Contents ⌂

Rounding and Estimating

Things to remember:

- If the next number is less than 5, round down.
- If the next number is 5 or more, round up.

- You may be asked to round to a specific place value (nearest 10, 2 decimal places, etc) or to a given number of significant figures.
- The first significant figure is the first non-zero digit. Every digit after this is significant.
- To estimate the answer to a calculation, round each number to one significant figure first.

Questions:

1.   Write 486 correct to the nearest ten.

........................490..............
(Total 1 mark)

2.   Write 73.564 correct to the nearest integer.

........................74..............
(Total 1 mark)

3.   Write 35.6791 correct to 1 decimal place.

........................35.7..............
(Total 1 mark)

4.   Write 2.24952 correct to 3 decimal places.

........................2.250..............
(Total 1 mark)

5.   Write 0.005286 correct to 2 significant figures.

........................0.0053..............
(Total 1 mark)

6.   Write 86 375 correct to 1 significant figure.

........................90 000..............
(Total 1 mark)

7.   Work out an estimate for the value of $\dfrac{32.7 \times 18.64}{3.609}$

$$\frac{30 \times 20}{4}$$

........................15..............
(Total 3 marks)

8

8.   Work out an estimate for the value of $\sqrt{9.21} + 2.75$

$\sqrt{9} + 3$

……………………… 6 ……………………
(Total 2 marks)

9.   Brian writes down the following

$$7.3 \times 2.69 = 196.37$$

Without doing the exact calculation, explain why Brian's answer cannot be correct.

$7 \times 3 = 21$
196.37 is nearly $10 \times$ bigger than 21

(Total 1 mark)

10.   Jaz gets paid £8.72 per hour she works.
Last week Jaz worked 38 hours.

a)   Work out an estimate for the amount Jaz got paid last week.

£9 × 40

……………… £360 ……………
(2)

b)   Is your answer to part (a) an underestimate or an overestimate?
You must explain your answer.

Overestimate as rounded both numbers up.

(1)
(Total 3 marks)

11.   A square has sides of 4.15 cm.

a)   Work out an estimate for the area of the square.

4^2

……………… 16 cm² ……………
(2)

b)   Is your answer to part (a) an underestimate or an overestimate?
You must explain your answer.

Underestimate as rounded down.

(1)
(Total 3 marks)

9

Error Intervals

Things to remember:

- Lower bound – the smallest value that a number can be (to a specified degree of accuracy).
- Upper bound – the largest value that a number can be (to a specified degree of accuracy).
- Error interval – the range of values in which the value could be (using inequality symbols).
- Make sure you read carefully what the specified degree of accuracy is or if the number has been truncated (chopped off!) instead of rounded.

Questions:

1. A church spire, s, measures as 46 metres, correct to the nearest metre.
   Write down the error interval for s.

   $45.5 \leq s < 46.5$

   (Total 2 marks)

2. Kevin, k, is 172 centimetres tall, correct to the nearest centimetre.
   Write down the error interval for k.

   $171.5 \leq k < 172.5$

   (Total 2 marks)

3. A number x is rounded to 2 significant figures.
   The result is 3.4
   Write down the error interval for x.

   $3.35 \leq x < 3.45$

   (Total 2 marks)

4. A number t is **truncated** to 1 decimal place.
   The result is 8.7
   Write down the error interval for t.

   $8.7 \leq t < 8.8$

   (Total 2 marks)

5. A bag of carrots, c, weighs 900 grams, to the nearest 100 grams.
   Write down the error interval for c.

   $850 \leq c < 950$

   (Total 2 marks)

6. Melanie used her calculator to work out the value of a number n.
   She wrote down the first three digits on her calculator.
   She wrote down 12.6
   Write down the error interval for n.

   $12.6 \leq n < 12.7$

   (Total 2 marks)

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Using a Calculator

Things to remember:

- You need to be familiar with your own scientific calculator ready for the exams.
- Buttons to learn (both where they are and what they do):

    - Brackets ( )
    - Powers x^2 $x^\blacksquare$ and roots $\sqrt{\blacksquare}$
    - Fractions and mixed numbers
    - Memory STO
    - Change or S→D S⇔D
    - Pi π
    - Sine, cosine and tangent sin cos tan

- Your calculator can do all sorts of other useful things too!

Questions:

1. Work out $(4.35 - 0.78)^2 + 2.056$
   Write down all the figures on your calculator display.

   14.8009

   (Total 2 marks)

2. Work out $\dfrac{4.3^3 - \pi}{9.67}$
   Write down all the figures on your calculator display.

   7.897146 572

   (Total 2 marks)

3. Work out $\sqrt{4.5^2 + 6.2^2 - 55.8\cos(73)}$
   Write down all the figures on your calculator display.

   6.509 658.891

   (Total 2 marks)

4. Work out $\dfrac{13.93 \times \sqrt[3]{30}}{1.3^4}$
   Write down all the figures on your calculator display.

   15.154 8436

   (Total 2 marks)

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Geometric Sequences

Things to remember:

- Each term is multiplied by the same number to get the next term. This number is called the common ratio.

Questions:

1. Which sequence is a geometric progression?
    Circle your answer.

    | 1 | 2 | 3 | 6 | | 1 | 2 | 3 | 4 |

    (1) 2 4 8 ×2 | 1 | 2 | 3 | 7 |

    (Total 1 mark)

2. Write down the next number in the sequence

    3   6   12   24

    ....**48**...................
    (Total 2 marks)

3. Write down the next number in the sequence

    2   6   18   54

    ....**162**...................
    (Total 2 marks)

4. Write down the next number in the sequence

    64   16   4   1

    ....$\frac{1}{4}$...................
    (Total 2 marks)

5. The first three terms of a number pattern are   1   2   4

    Gabriel says the first five terms of this number pattern are   1   2   4   8   16

    a)   Write down the rule that Gabriel could have used to get the 4th and 5th terms.

    ....**Multiply by 2**...................
    (1)

    b)   Write down the 6th term of Gabriel's number pattern.

    ....**32**...................
    (1)
    (Total 2 marks)

Parallel Graphs

Things to remember:

- The general equation for a linear graph is $y = mx + c$, where m is the gradient and c is the y-intercept.
- Parallel graphs have the same gradient.

Questions:

1.   Here are the equations of four straight lines.

$y = 3x + 4$ ⟵ (circled)

$y = 2x + 3$

$y = \frac{1}{3}x + 3$

$y = -\frac{1}{3}x + 4$

One of the graphs is parallel to $y = 3x - 4$
Circle the correct answer.

(Total 1 mark)

2.   Write down the equation of a line parallel to $y = 5x - 6$

$y = 5x \pm \boxed{}$

(Total 1 mark)

3.   Write down the equation of a line parallel to $y = 2x + 8$

$y = 2x \pm \boxed{}$

(Total 1 mark)

4.   Here are the equations of four straight lines.

$8 = 2x - y$ $y = 2x - 8$

$y = 2x - 8$ (circled)

$2x + y - 8 = 0$ $y = -2x + 8$

$8 = 2x + y$

Two of these lines are parallel.
Circle the two parallel lines.

(Total 1 mark)

Writing Equations of Linear Graphs

Things to remember:

- The general equation for a linear graph is $y = mx + c$, where m is the gradient and c is the y-intercept.
- Given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, $m = \frac{y_2 - y_1}{x_2 - x_1}$
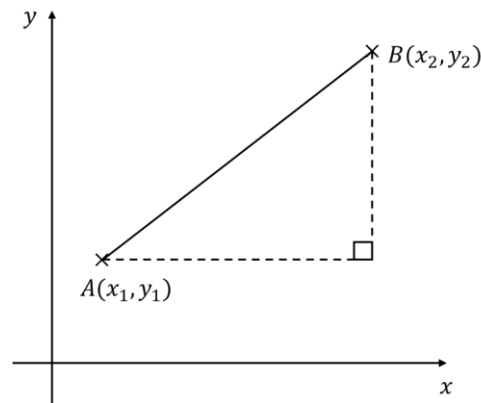
Questions:

1. A line passes through the point $(0, 6)$
   The gradient of this line is 3
   Write down the equation of this line.

   ....$y = 3x + 6$.............
   (Total 2 marks)

2. A line passes through the point $(0, -1)$
   The gradient of this line is 2
   Write down the equation of this line.

   ....$y = 2x - 1$.............
   (Total 2 marks)

3. A straight line has equation $y = x + 5$

   a) Write down the gradient of the line.

   ....1.............
   (1)

   b) Write down the coordinates of the point where the line crosses the y-axis.

   ....$(0, 5)$.............
   (1)
   (Total 2 marks)

4. A straight line has equation $y = 2 - 3x$

   a) Write down the gradient of the line.

   ....-3.............
   (1)

   b) Write down the coordinates of the point where the line crosses the y-axis.

   ....$(0, 2)$.............
   (1)
   (Total 2 marks)

14

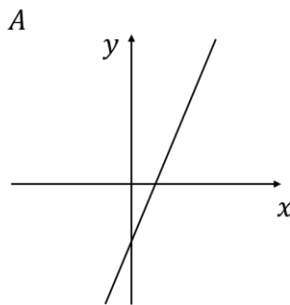5.   A straight line has equation $3y - 6x = 12$

$3y = 6x + 12$
$y = 2x + 4$

a)   Work out the gradient of the line.

..................................2..........................
                                                            (2)

b)   Write down the coordinates of the point where the line crosses the y-axis.

..................$(0, 4)$..................
                                                            (1)
                                        (Total 3 marks)

6.   The diagrams show four graphs.
     Match each graph to its equation.

A

B
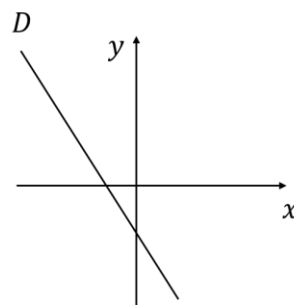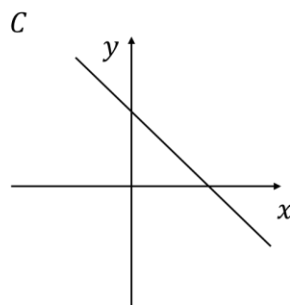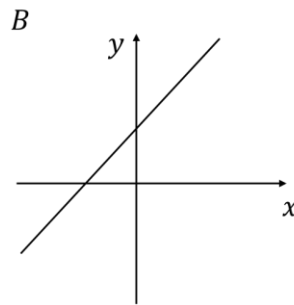
Equation	Graph
$y = 4 - x$	C
$y = -2x - 3$	D
$y = 2x - 5$	A
$y = x + 3$	B

C

D

                                        (Total 4 marks)

7.   Write down the equation of the line parallel to $y = 5x - 2$ passing through the point $(0, 4)$

..................$y = 5x + 4$..................
                                        (Total 2 marks)

8.   Write down the equation of the line parallel to $y = 2x + 3$ passing through the point $(3, 7)$
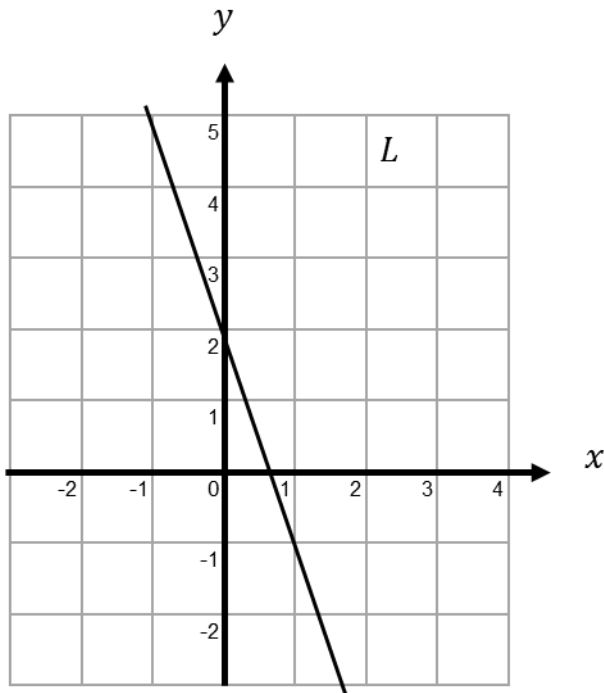
$y = 2x + c$ $(3, 7)$
$7 = 2 \times 3 + c$
$1 = c$

..................$y = 2x + 1$..................
                                        (Total 3 marks)

15

9.   Find the equation of line L.
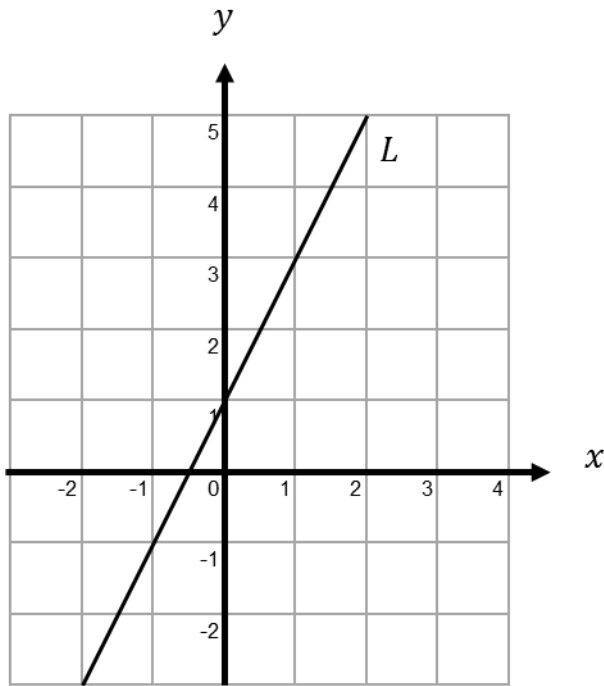
$y = 2x + 1$

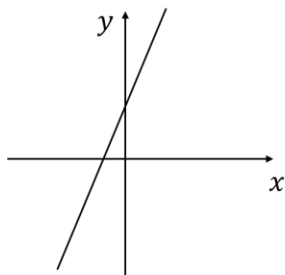(Total 3 marks)

10.   Find the equation of line L.

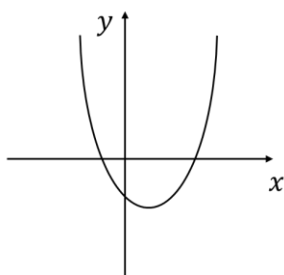$y = -3x + 2$

(Total 3 marks)

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Sketching Nonlinear Graphs
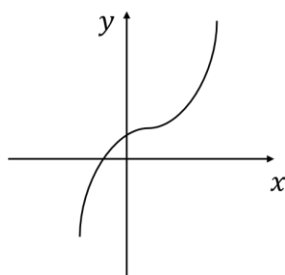
Things to remember:

- To fill in the table of values, substitute the x values into the equation of the graph to find the corresponding y values.
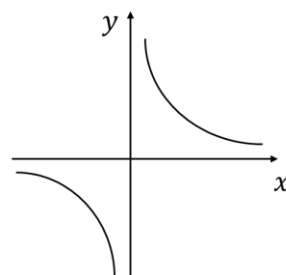- Plot the coordinates and join them with a smooth curve.
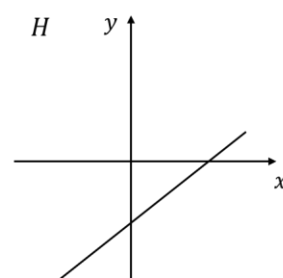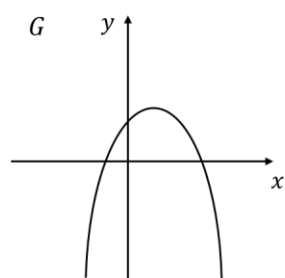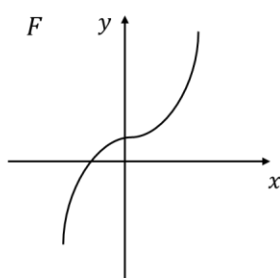

| Linear | Quadratic | Cubic | Reciprocal |

Questions:

1. Here are eight graphs.
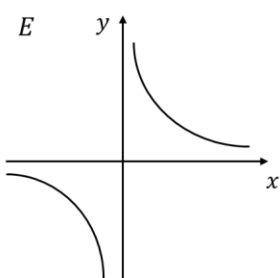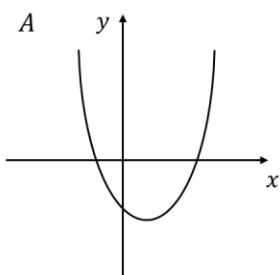
Write down the letter of the graph that could have the equation:

i) $y = 2x + 4$

.... D
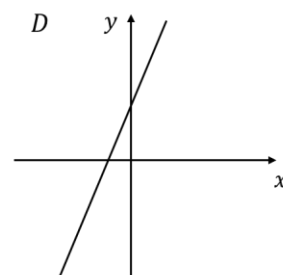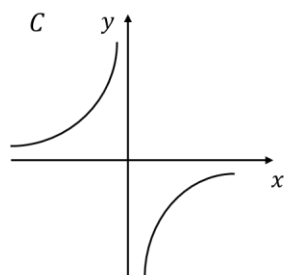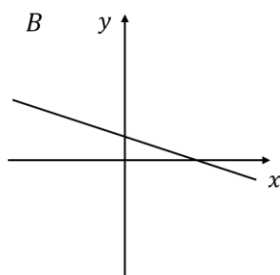
ii) $y = \dfrac{2}{x}$

.... E

iii) $y = x^2 - 2x - 3$

.... A

iv) $y = x^3 + 2$

.... F

(Total 4 marks)

2. a) Complete the table of values for $y = x^2 - x - 6$

x	-3	-2	-1	0	1	2	3
y	6	○	-4	-6	-6	-4	○

(2)

b) On the grid draw the graph of $y = x^2 - x - 6$ for values of x from -3 to 3
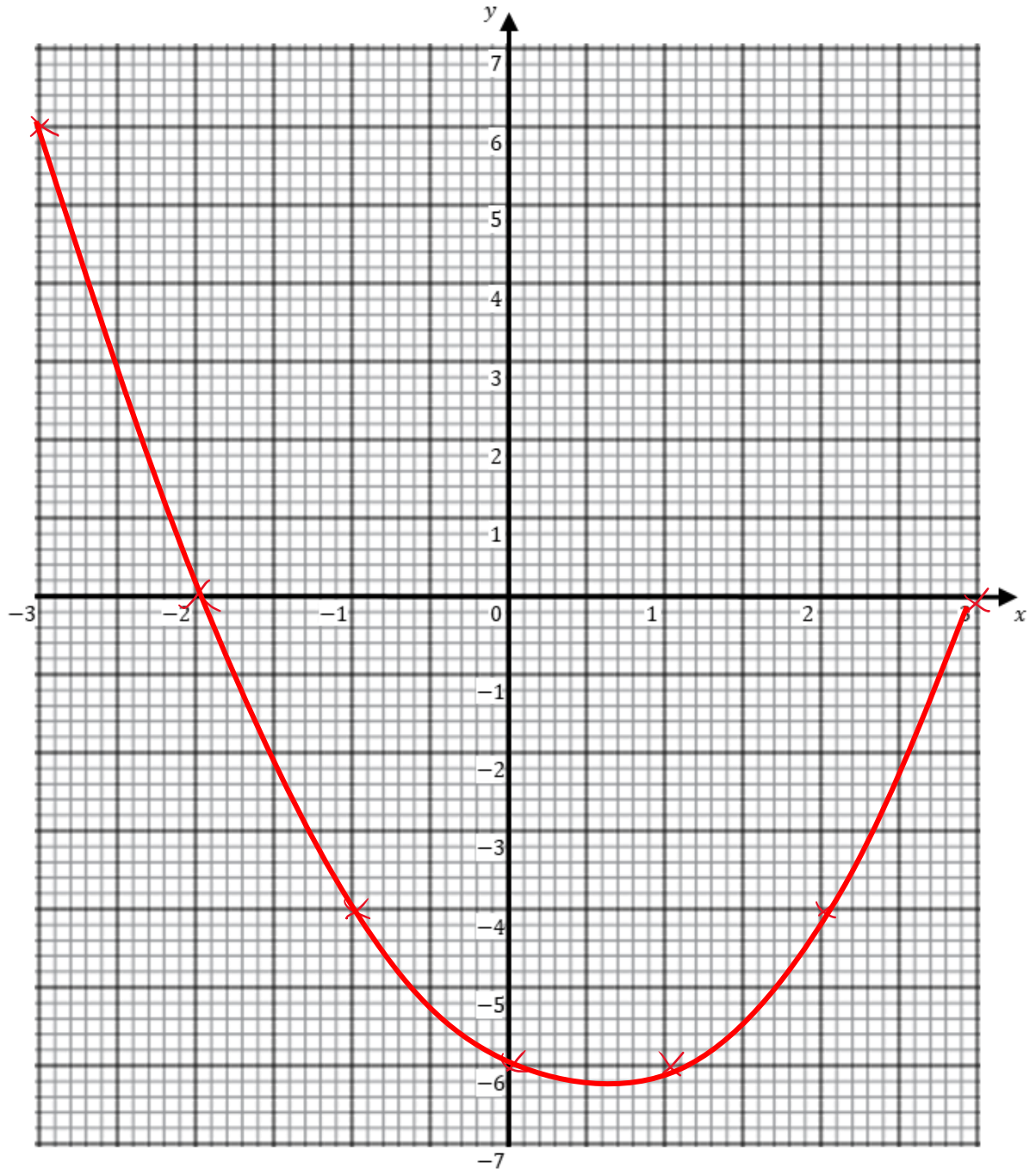
(2)

(Total 4 marks)

3. a) Complete the table of values for $y = \frac{2}{x}$

x	0.2	0.5	1	2	4	8
y	10	4	2	1	0.5	0.25

(2)

b) On the grid draw the graph of $y = \frac{2}{x}$

(2)
(Total 4 marks)

19
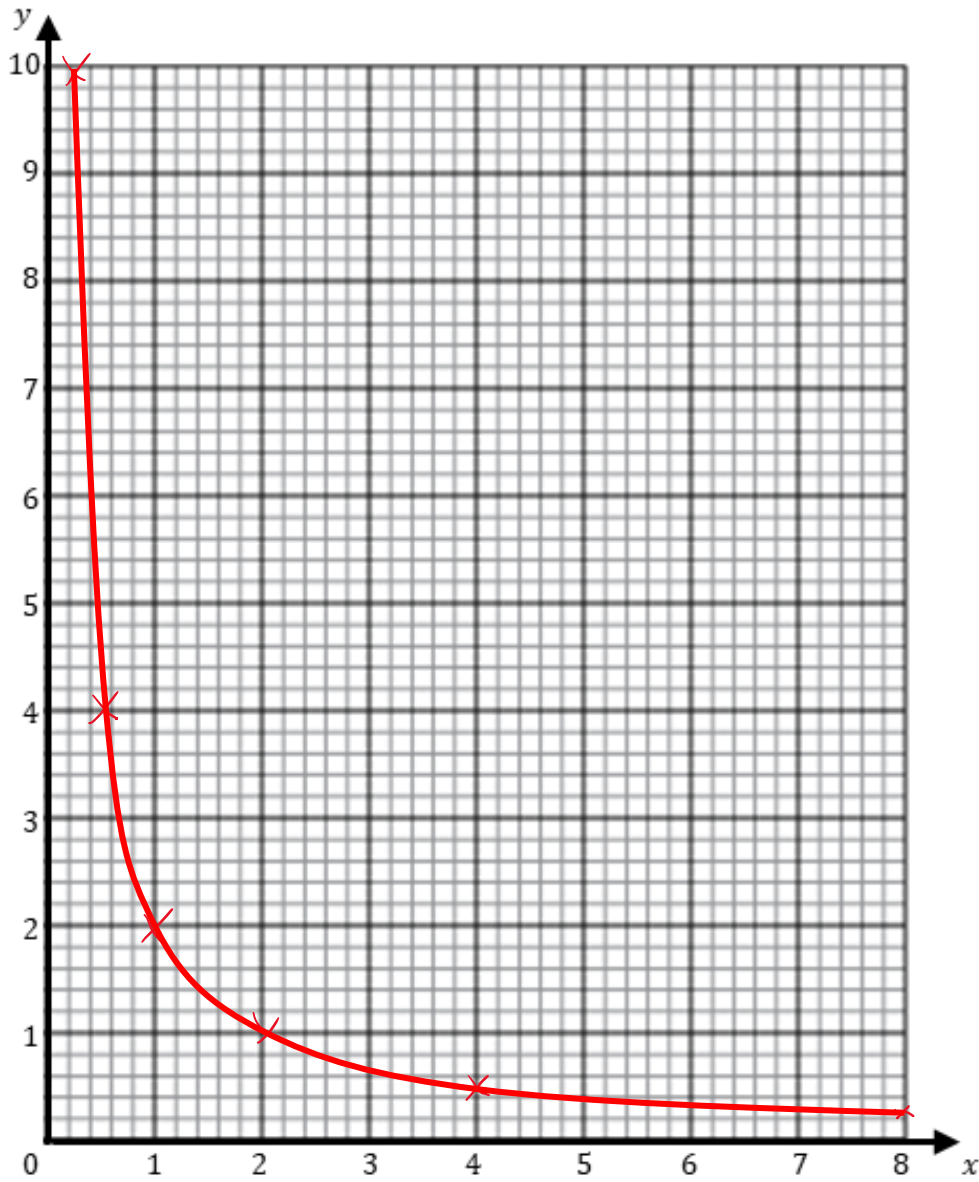
4. a) Complete the table of values for $y = x^3 + x - 2$

x	-2	-1	0	1	2
y	-12	-4	-2	0	8

(2)

b) On the grid draw the graph of $y = x^3 + x - 2$ for values of x from -2 to 2

(2)

(Total 4 marks)

20
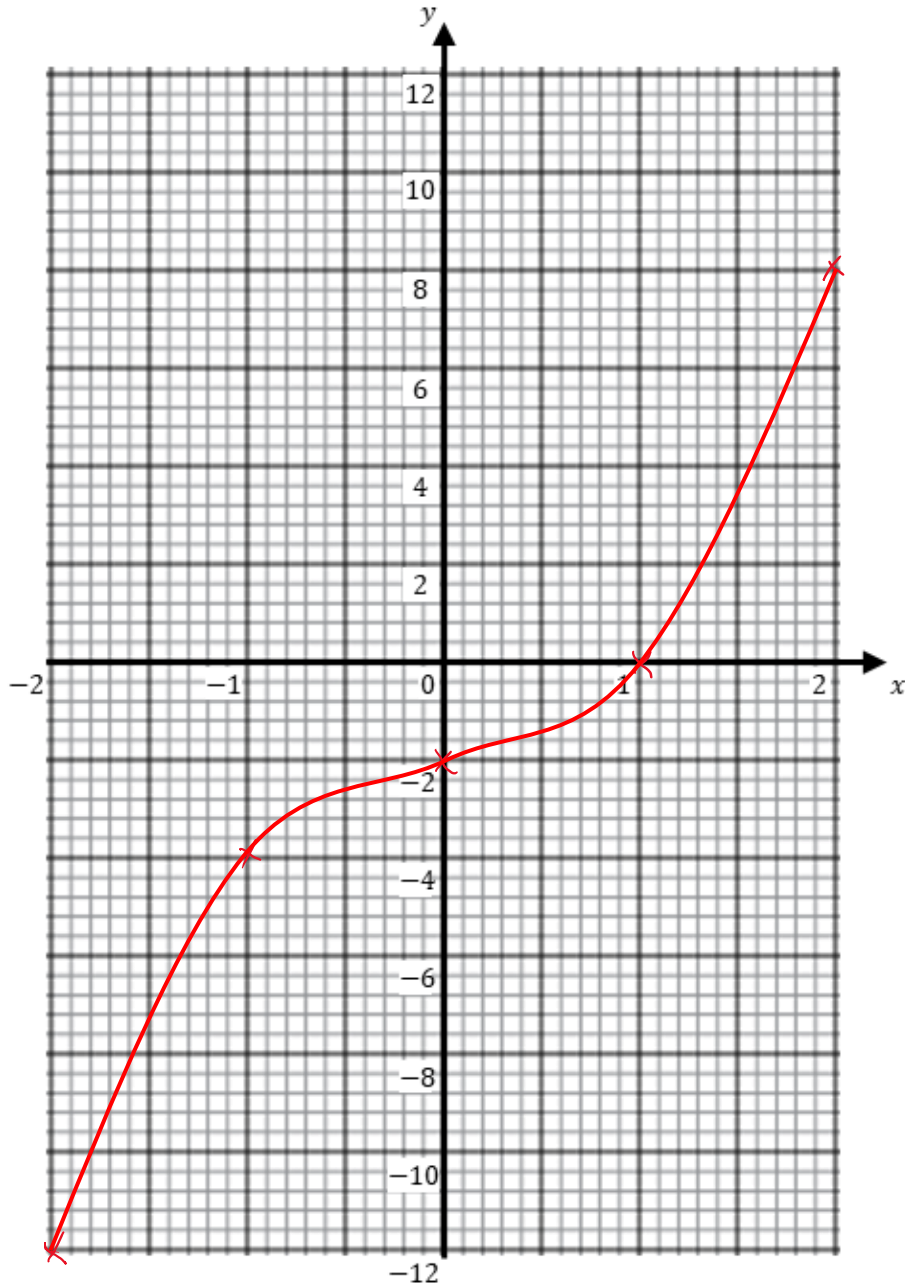
Set Notation

Things to remember:

Universal set (everything)

Elements

$$\xi = \{a, b, c, d, e, f, g, h\}$$

Braces
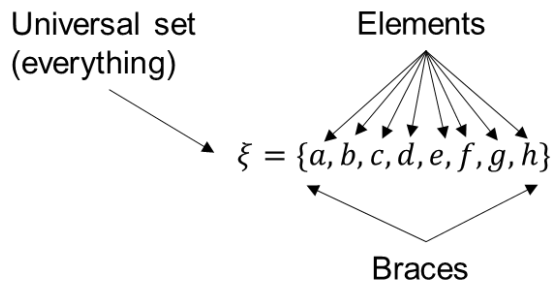
$$A = \{a, c, d, e\}$$

$$B = \{b, c, e, f, h\}$$
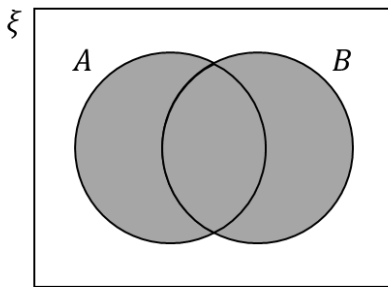
$A \cup B$
A "or" B
Union
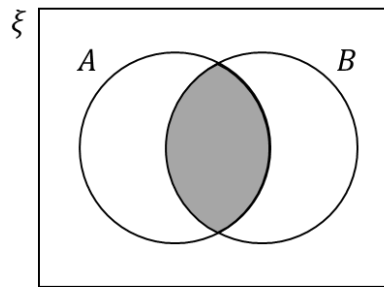
$A \cap B$
A "and" B
Intersection

A' means "not A" – the complement of A

Questions:

1.   Here is a Venn diagram.
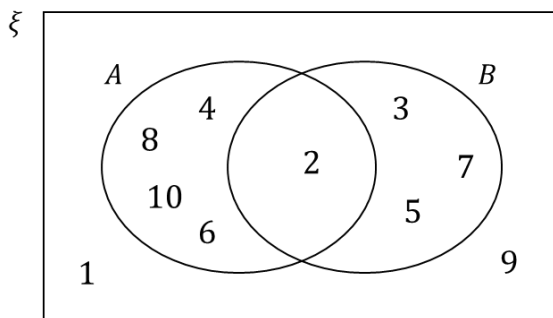
Write down the items that are in set

(i) A

(ii) A'

(iii) $A \cap B$

2, 4, 6, 8, 10

1, 3, 5, 7, 9

2

(Total 3 marks)

2. Here is a Venn diagram.
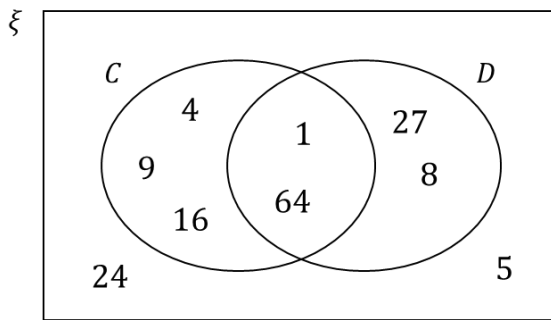
Write down the items that are in set

(i) C

(ii) $C \cup D$

(iii) $C' \cap D$

1, 4, 9, 16, 64

1, 4, 8, 9, 16, 27, 64

8, 27

(Total 3 marks)

3. Here is a Venn diagram.
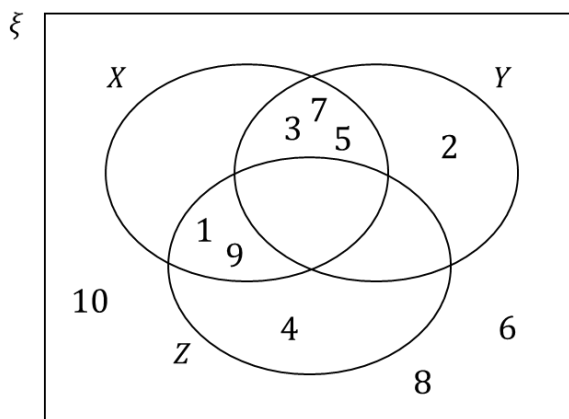
Write down the items that are in set

(i) X

(ii) Y

(iii) Z'

(iii) $X \cap Y$

(iii) $X \cup Y \cup Z$

1, 3, 5, 7, 9

2, 3, 5, 7

2, 3, 5, 6, 7, 8

3, 5, 7

1, 2, 3, 4, 5, 7, 9

(Total 5 marks)

Contents ⌂

Solving Linear Equations

Things to remember:

- "Solve" means to find the value of the unknown (what number the letter represents).
- The inverse of + is − and the inverse of × is ÷
- Work one step at a time, keeping your = signs in line on each new row of working.
- You MUST think about the order in which the operations act on the unknown and do the reverse.
- If there are brackets in the question, multiply out the brackets before solving the equation.
- Where the unknown appears twice (on both sides of the = sign) start by eliminating the smallest unknown first.

Questions:

1.　Solve

a)　$5x + 3 = 33$

$5x = 30$

$x = \dots\dots\dots 6 \dots\dots\dots$

(2)

b)　$3(y + 6) = 24$

$3y + 18 = 24$

$3y = 6$

$y = \dots\dots\dots 2 \dots\dots\dots$

(2)

c)　$4m + 7 = 2m + 13$

$2m = 6$

$m = \dots\dots\dots 3 \dots\dots\dots$

(3)

d)　$3(t + 4) = 5t + 4$

$3t + 12 = 5t + 4$

$8 = 2t$

$t = \dots\dots\dots 4 \dots\dots\dots$

(3)

(Total 10 marks)

23

2.   Solve

a) $7w + 3 = 5w + 9$

$2w = 6$

$w = \underline{\qquad 3 \qquad}$

(3)

b) $7y + 6 = 5(y - 2)$

$7y + 6 = 5y - 10$

$2y = -16$

$y = \underline{\qquad -8 \qquad}$

(3)

(Total 6 marks)

3.   Solve $4(2k - 5) = 5k + 4$

$8k - 20 = 5k + 4$

$3k = 24$

$k = \underline{\qquad 8 \qquad}$

(Total 3 marks)

4.   Solve $2(2r + 1) = 3(r - 4)$

$4r + 2 = 3r - 12$

$r = -14$

$r = \underline{\qquad -1 \qquad}$

(Total 3 marks)

Expanding and Factorising Quadratics

Things to remember:

- When expanding brackets, everything in the first bracket must multiply everything in the second bracket.
- Either use a multiplication grid or FOIL (First, Outside, inside, Last)
- You will need to simplify your answer (collect like terms)

- For any quadratic $x^2 + ax + b$, find a pair of numbers with a sum of a and a product of b in order to factorise.
- Be really careful with negatives when you're looking at the sums and products!
- Again, a multiplication grid will help.

Questions:

1. a) Expand and simplify $(x + 2)(x + 3)$

$$x^2 + 2x + 3x + 6$$

$$\underline{x^2 + 5x + 6}$$
(2)

b) Factorise $x^2 + 4x + 3$

$$\underline{(x + 1)(x + 3)}$$
(2)
(Total 4 marks)

2. Expand and simplify $(m + 3)(m - 4)$

$$m^2 + 3m - 4m - 12$$

$$\underline{m^2 - m - 12}$$
(Total 2 marks)

3. Expand and simplify $(5x - 2)(2x + 3)$

$$10x^2 - 4x + 15x - 6$$

$$\underline{10x^2 + 11x - 6}$$
(Total 2 marks)

4. Factorise $x^2 - x - 6$

$(x-3)(x+2)$
(Total 2 marks)

5. a) Factorise $x^2 + 8x + 12$

$(x+2)(x+6)$
(2)

b) Hence, or otherwise, solve $x^2 + 8x + 12 = 0$

$x = -2 \text{ or } -6$
(2)
(Total 2 marks)

6. Expand and simplify $(2x - 6)(2x + 6)$

$4x^2 - 12x + 12x - 36$

$4x^2 - 36$
(Total 2 marks)

7. Factorise $x^2 - 81$

$(x+9)(x-9)$
(Total 2 marks)

Rearranging Formulae (Changing the Subject)

Things to remember:

- Identify which variable needs to be on its own.
- Then move all terms that contain that variable to one side of the formula. Remember to move **all** terms if the variable appears in more than one.
- Next separate out the required letter on its own.
- The inverse of + is − and the inverse of × is ÷
- Work one step at a time, keeping your = signs in line on each new row of working.

Questions:

1. Make t the subject of the formula $v = 5t + 7$

    $v - 7 = 5t$

    $t = \dfrac{v-7}{5}$

    (Total 2 marks)

2. Make h the subject of the formula $G = 2h - 8$

    $G + 8 = 2h$

    $h = \dfrac{G+8}{2}$

    (Total 2 marks)

3. Make p the subject of the formula $q = \frac{p}{r} - s$

    $q + s = \dfrac{p}{r}$

    $p = r(q+s)$

    (Total 2 marks)

4. Make k the subject of the formula $m = \frac{k}{2} - 3n$

    $m + 3n = \dfrac{k}{2}$

    $2(m+3n)$

    (Total 2 marks)

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Linear Simultaneous Equations (Including Graphs)

Things to remember:

Step 1 → Scale up each term in one or both equations to make the coefficients the same for either the x terms or the y terms.
Step 2 → Add or subtract to eliminate the common coefficients (subtract if the signs in front of these are the same, add if the signs in front of these are different.)
Step 3 → Solve the equation to find the value of the first unknown.
Step 4 → Substitute and solve to find the value of the second unknown.

Questions:

1.   Solve the simultaneous equations $4x - y = 13$
$\quad 2x - 3y = 4 \quad \times 2$

$4x - y = 13$
$-\ 4x - 6y = 8$
$\overline{5y = 5}$
$y = 1$

$4x - y = 13$
$4x - 1 = 13$
$4x = 14$
$x = 3.5$

$\underline{x = 3.5, \ y = 1}$
(Total 3 marks)

2.   Solve the simultaneous equations $4x + 2y = 10 \quad \times 5$
$\quad 5x + 3y = 12 \quad \times 4$

$20x + 10y = 50$
$-\ 20x + 12y = 48$
$\overline{-2y = 2}$
$y = -1$

$4x + 2y = 10$
$4x - 2 = 10$
$4x = 12$
$x = 3$

$\underline{x = 3, \ y = -1}$
(Total 3 marks)

3.   Solve the simultaneous equations $3x - y = -4 \quad \times 3$
$\quad 2x - 3y = 9$

$9x - 3y = -12$
$-\ 2x - 3y = 9$
$\overline{7x = -21}$
$x = -3$

$3x - y = -4$
$-9 - y = -4$
$y = -5$

$\underline{x = -3, \ y = -5}$
(Total 3 marks)

4. In a shop 3 coffees and 2 pastries cost £10.80
In the same shop 4 coffees and 1 pastry cost £12.40
Work out the price for one coffee and the price for one pastry.

$3c + 2p = 10.8$

$4c + p = 12.4$ (×2)

$3c + 2p = 10.8$

$- 8c + 2p = 24.8$

$-5c = -14$

$c = 2.8$

$3c + 2p = 10.8$

$8.4 + 2p = 10.8$

$2p = 2.4$

$p = 1.2$

Coffee : £2.80
Pastry : £1.20

(Total 4 marks)

5. The graphs of the straight lines with equations $3x + y = 4$ and $3y - 2x = 1$ have been drawn on the grid.
Use the graphs to solve the simultaneous equations $3x + y = 4$
$3y - 2x = 1$
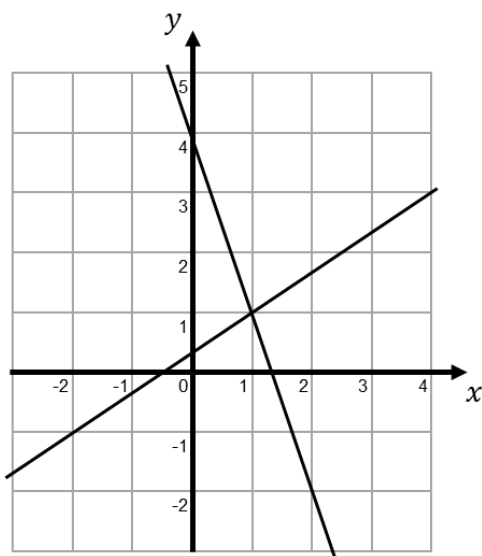
$x = 1, y = 1$
....
(Total 2 marks)

Solving Linear Inequalities

Things to remember:

- Solve inequalities in the same way you solve equations.
- If you multiply or divide by a negative, you will need to change the direction of the inequality symbol.
- An integer is a whole number.
- On a number line, use a full circle to show a value can be equal, and an empty circle to show it cannot.

< means less than
> means greater than
≤ means less than or equal to
≥ means greater than or equal to

Questions:

1. a) Solve the inequality $4x - 6 < 10$

$$4x < 16$$

..........................$x < 4$.........................
(2)

b) On the number line, represent the solution set to part (a)

(1)
(Total 3 marks)

2. n is an integer such that $2 < 2n + 4 < 10$
Write down all the possible values of n

$$-2 < 2n < 6$$
$$-1 < n < 3$$

.............$0, 1, 2$..............
(Total 4 marks)

3. Solve the inequality $7x - 2 \leq 26$

$$7x \leq 28$$
$$x \leq 4$$

..........................$x \leq 4$..........................
(Total 2 marks)

4. Write down the inequality shown on the number line.
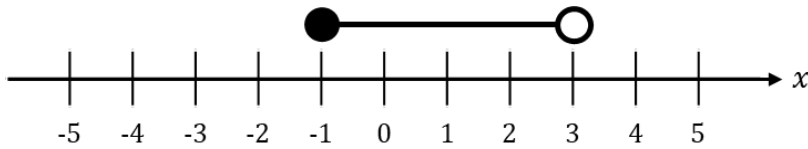
$$-1 \leq x < 3$$

(Total 2 marks)

5. Solve $3x - 6 \geq 15$

$$3x \geq 21$$
$$x \geq 7$$

$$x \geq 7$$

(Total 2 marks)

6. Solve the inequality $2(n + 4) < 8$

$$2n + 8 < 8$$
$$2n < 0$$
$$n < 0$$

$$n < 0$$

(Total 2 marks)

7. Solve $2y + 5 > 4y - 9$

$$14 > 2y$$
$$7 > y$$

$$y < 7$$

(Total 3 marks)

8. a) Solve the inequality $-3 < 2x + 5 \leq 13$

$$-8 < 2x \leq 8$$
$$-4 < x \leq 4$$

$$-4 < x \leq 4$$

(3)

b) On the number line, represent the solution set to part (a)
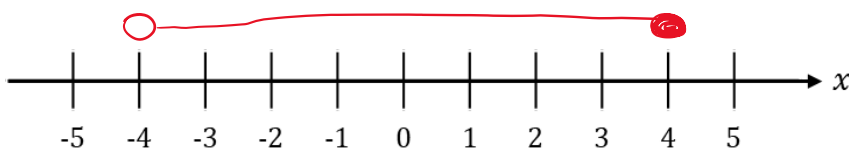
(1)

(Total 4 marks)

Properties of Circles

Things to remember:
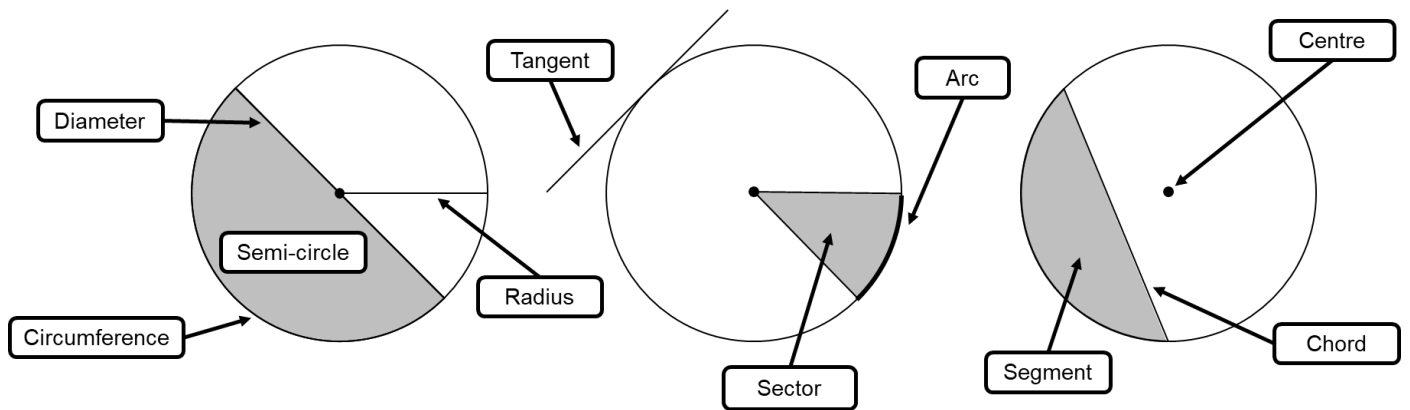
Questions:

1.   Draw the parts on the circles below
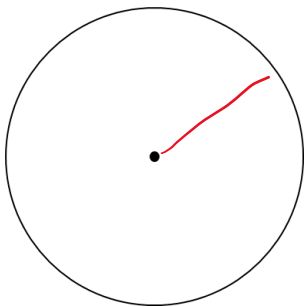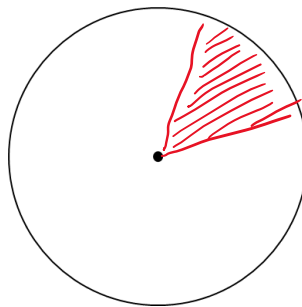
   (i)   Radius   (ii)   Sector   (iii)   Chord
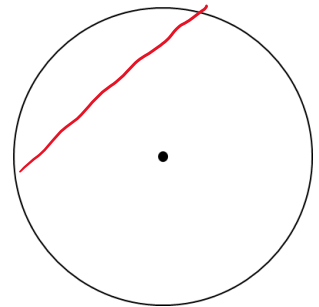
(Total 3 marks)

2.   Write down the mathematical name for the straight line touching the circle.
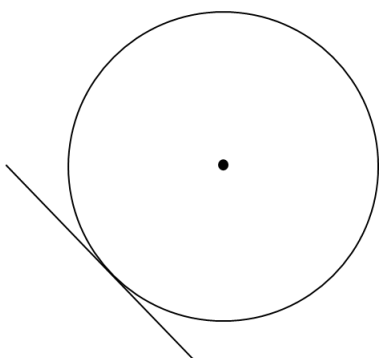
..........Tangent................
(Total 1 mark)

Congruence and Similarity

Things to remember:

- Congruent shapes have all sides and angles equal.
- Similar shapes have all angles equal but one is an enlargement of the other.

Questions:

1. Here are some rectangles on a grid of centimetre squares.
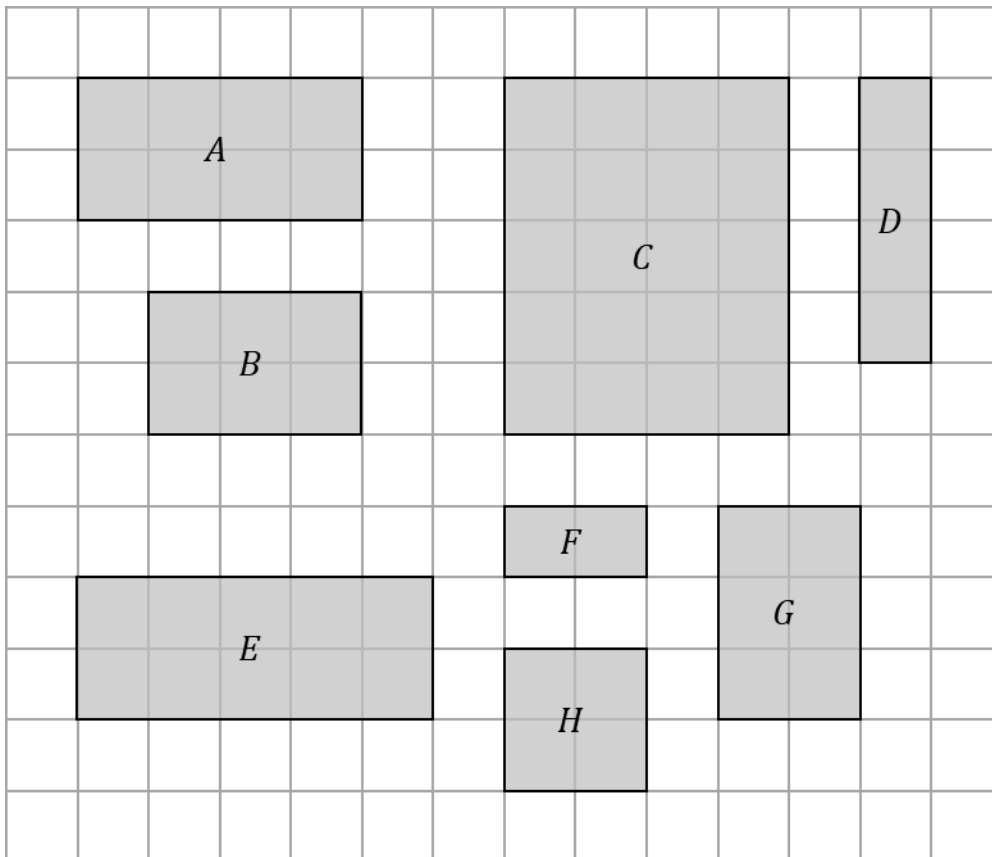
a) Write the letter of the rectangle that is similar to rectangle *F*

............................A.................................
(1)

b) Write the letters of two rectangles that are congruent.

......................B............................ and ....G......................
(1)
(Total 2 marks)

2. Triangles A and B are similar.
   Tick the correct boxes in the table below.

	True	False	Maybe
Triangles A and B have different size angles		✓	
Triangle A has a larger area than triangle B			✓
If triangle A is scalene, triangle B has to be scalene	✓		

(Total 3 marks)

3. Here are some shapes on a grid of centimetre squares.
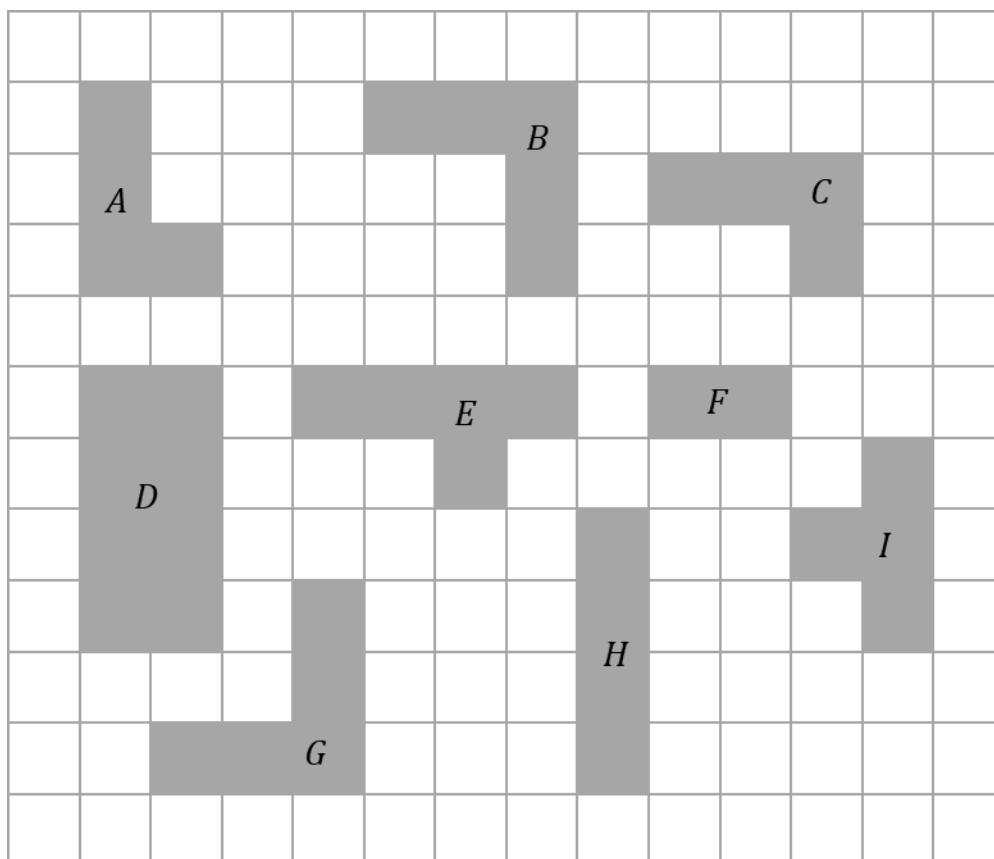
a) Write the letter of the shape that is similar to shape F

.... D ....
(1)

b) Write the letter of the shape that is congruent to shape A

.... C ....
(1)

b) Write the letters of two other shapes that are congruent.

.... B .... and .... G ....
(1)
(Total 3 marks)

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Loci and Construction

Things to remember:

- You may be asked to "construct", "use a ruler and compasses" or "show all your construction lines" – your working out must always be clear.
- Bisect means "cut in half".
- Sometimes you will have to shade a particular region – make sure you read the question carefully.

Questions:

1. Use ruler and compasses to construct the bisector of angle BAC.
   You must show all your construction lines.
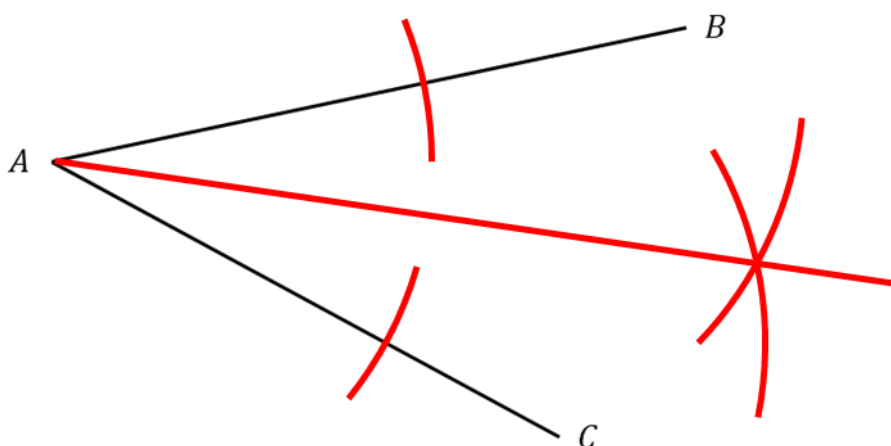
(Total 2 marks)

2. Use ruler and compasses to construct a perpendicular bisector of the line AB.
   You must show all your construction lines.
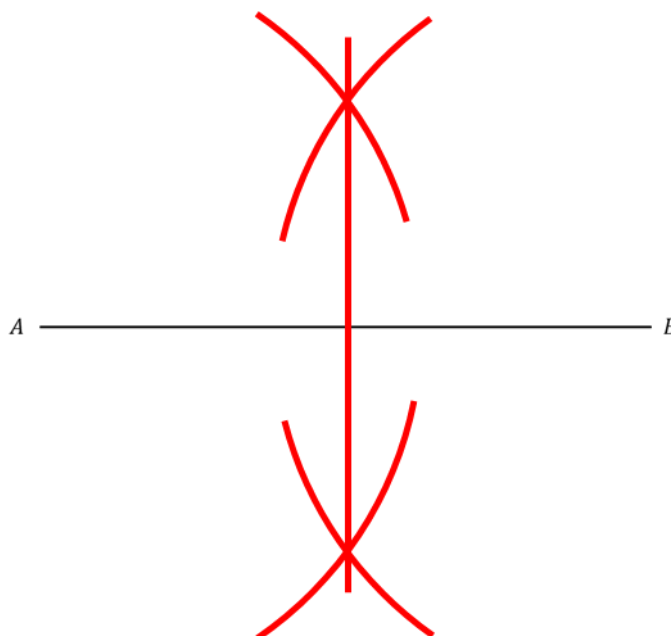
(Total 2 marks)

3. Use ruler and compasses to construct the perpendicular from point C to the line AB.
    You must show all your construction lines.
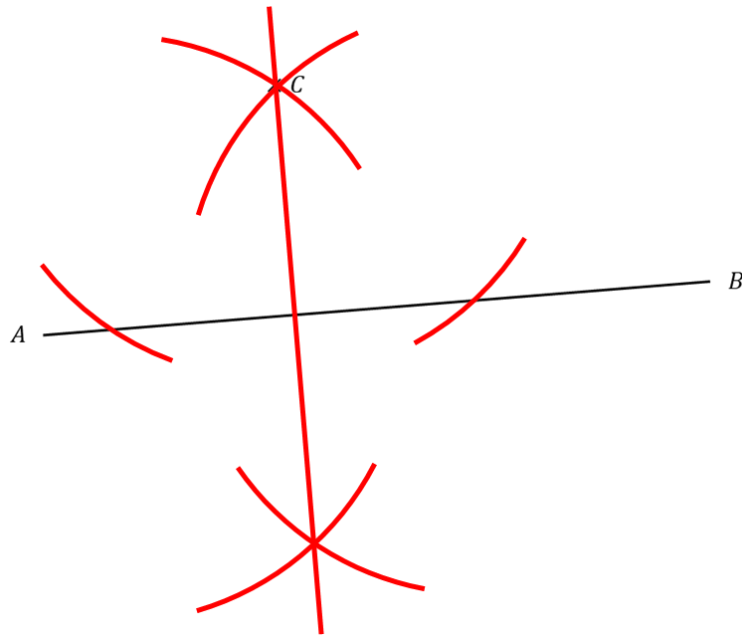
(Total 2 marks)

4. In the space below, use a ruler and compasses to construct a $60°$ angle.
    You must show all your construction lines.
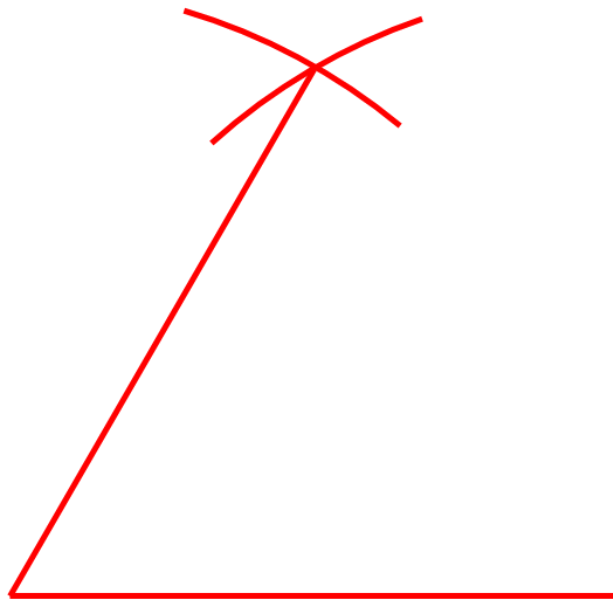
(Total 4 marks)

Contents ⌂

5.   A power station is going to be built.
      It must be more than 14 km from town A and no more than 10 km from town B.
      1 cm represents 2 km
      Shade the region on the diagram where the power station can be built.
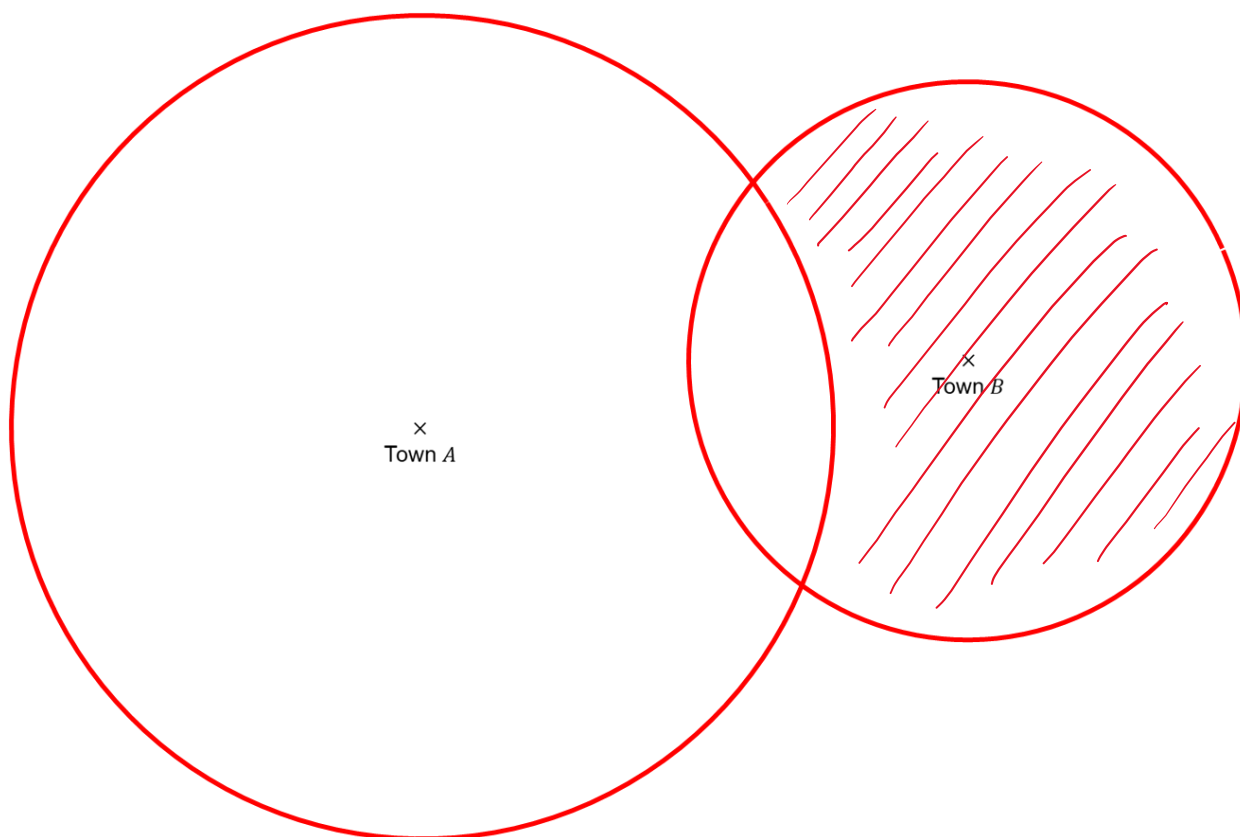
                                                                    (Total 3 marks)

6.   Here is a scale drawing of a garden.
      The scale is 1 cm to 2 m
      A cherry tree is going to be planted.
      The tree must be more than 4 m from the patio.
      The tree must be more than 7 m from the pond.
      Shade the region where the tree can be planted.
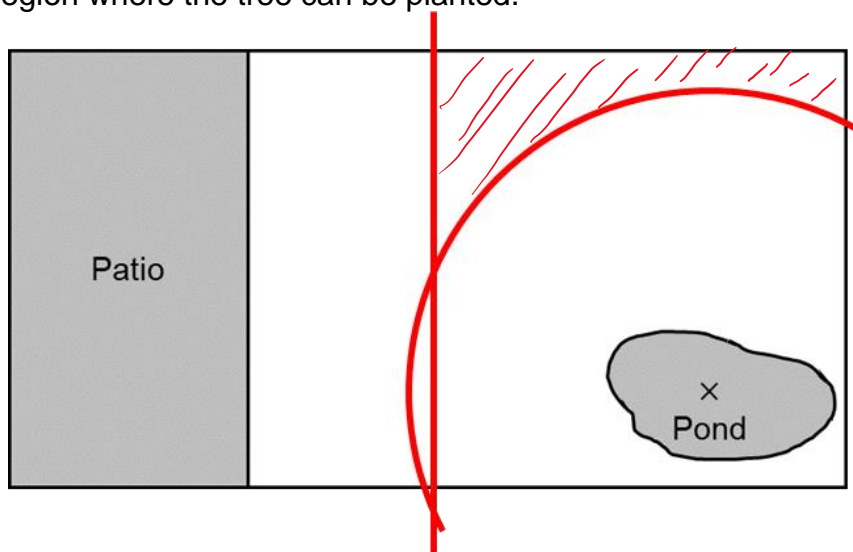
                                                                    (Total 3 marks)

Bearings

Things to remember:

- Always measure bearing <u>clockwise</u> from the <u>North</u> line and give your answer <u>3 digits</u>.
- If the diagram is drawn accurately, use the given scale.

Questions:

1.
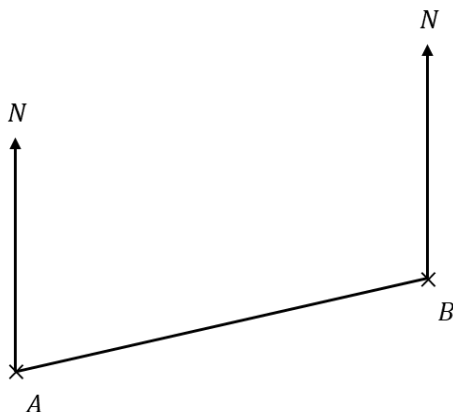
a)   Measure the bearing of B from A.

................................077................................°

(1)

b)   Measure the bearing of A from B.

................................257................................°

(1)

(Total 2 marks)

2.   The accurate scale drawing shows the positions of boat A and boat B.
Boat C is on a bearing of 100° from A.
Boat C is on a bearing of 250° from B.
On the diagram, mark with a cross (✗) the position of boat C on the diagram.

C
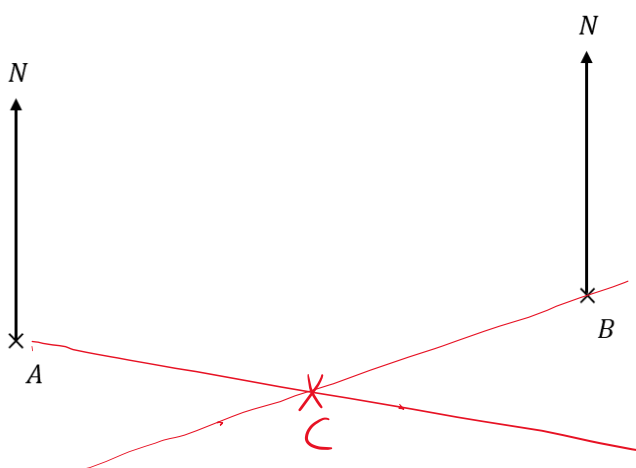
(Total 3 marks)

Transformations

Things to remember:

- Reflection – the shape is flipped in a mirror line
- Rotation – the shape is turned a number of degrees, around a centre, clockwise or anti-clockwise
- Translation – the shape is moved by a vector $\begin{pmatrix} x \\ y \end{pmatrix}$
- Enlargement – the shape is made bigger or smaller by a scale factor from a centre.

Questions:

1.
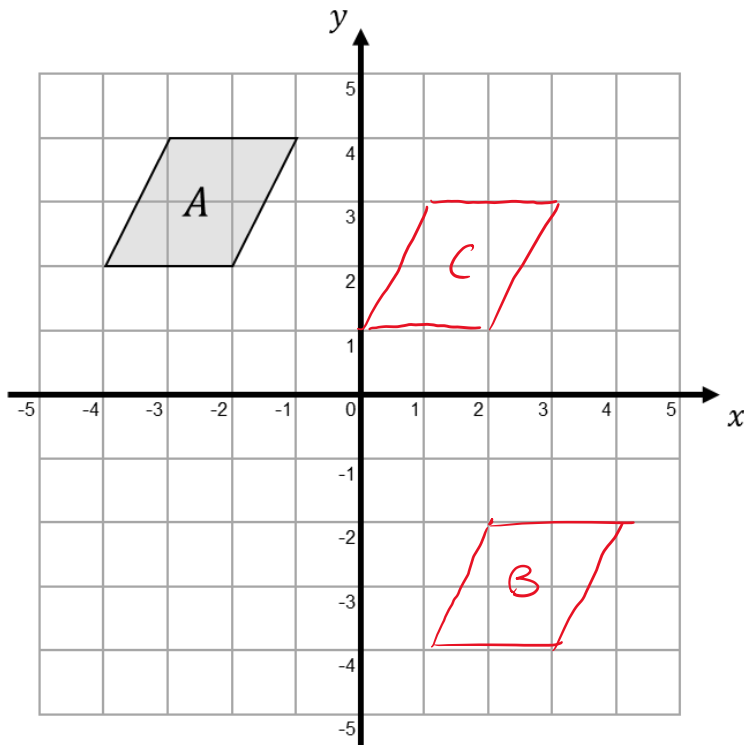
    a)   Rotate shape A by 180°, centre $(0,0)$. Label it B.

(2)

    b)   Translate shape A by the vector $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$. Label it C.

(2)
(Total 4 marks)

2.
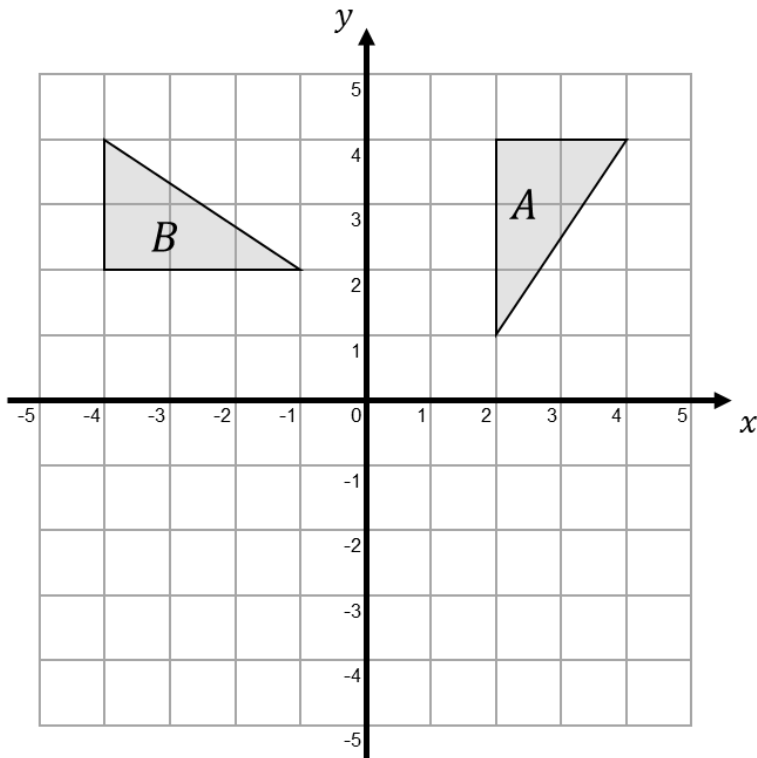
Describe fully the single transformation that maps triangle A on triangle B.

Rotated 90° anti-clockwise about (0,0)

(Total 2 marks)

3.
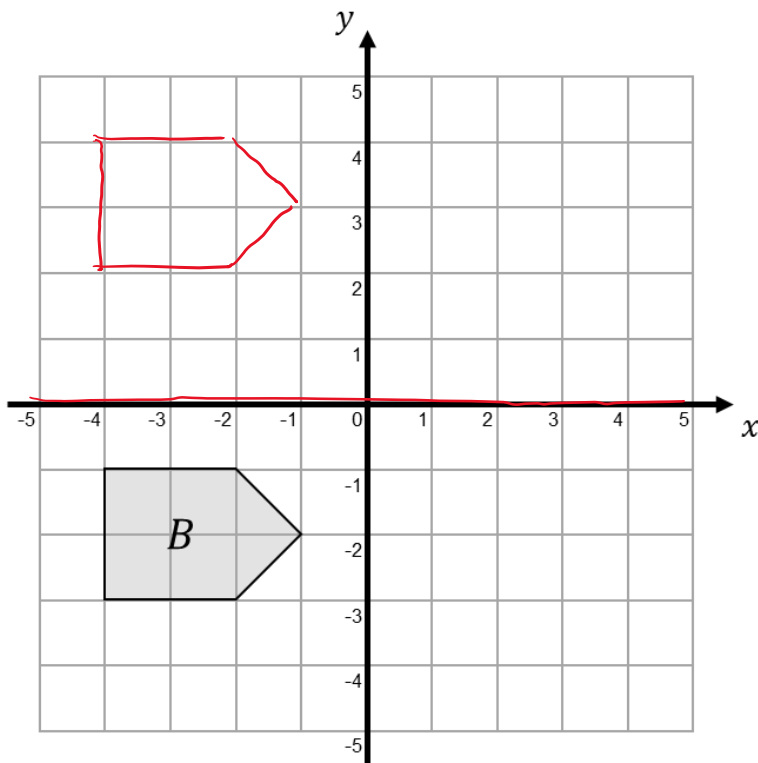
Reflect shape B in the line $y = 1$

(Total 2 marks)

4.
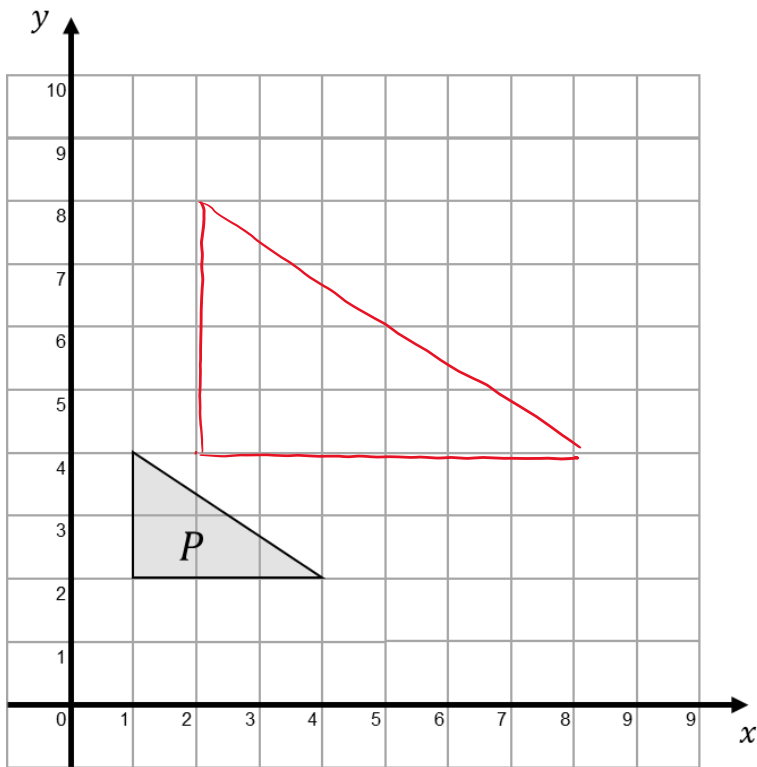
Enlarge shape P by scale factor 2 from centre $(0, 0)$

(Total 2 marks)

5.
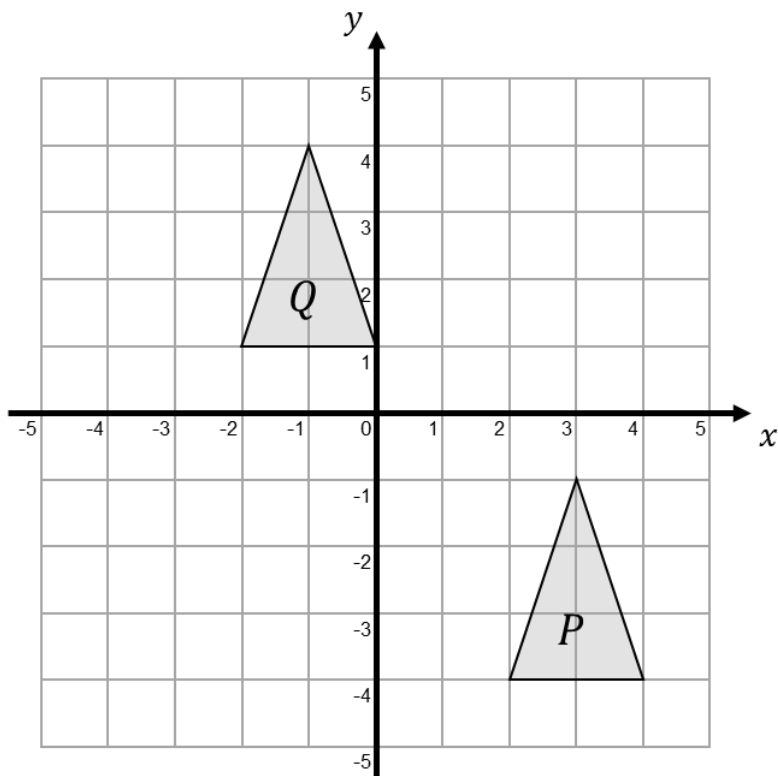
Describe fully the single transformation that maps triangle P on triangle Q.

Translated by vector $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$

(Total 2 marks)

Trigonometry

Things to remember:

1. Label the sides opposite, adjacent and hypotenuse
2. Substitute the values into:

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

3. Solve the equation! (You will need your calculator for these questions)
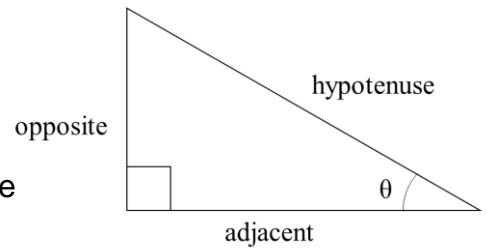
Questions:

1. Diagram NOT accurately drawn
   Work out the value of p.
   Give your answer correct to 1 decimal place.

   $7 \times \tan 39$

   $= 5.6684...$

   p cm — O
   39°
   7 cm A
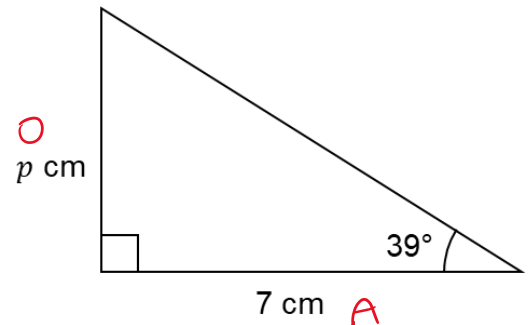
   ....5.7................. cm
   (Total 3 marks)

2. Diagram NOT accurately drawn
   Work out the value of x.
   Give your answer correct to 1 decimal place.

   $5.4 \times \cos 42$

   $= 4.0129...$

   x m A
   42°
   5.4 m
   H

   ....4.0............... m
   (Total 3 marks)

42
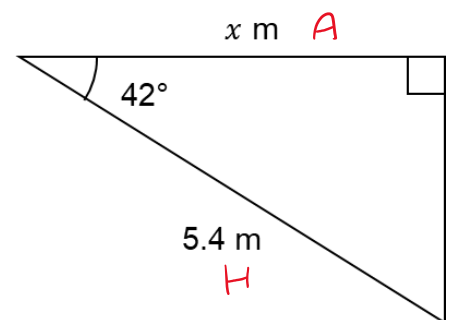
3. Diagram NOT accurately drawn
Work out the value of x.
Give your answer correct to 1 decimal place.

$\sin^{-1}\left(\dfrac{4.2}{6.8}\right)$

$= 38.1445\ldots$

O 4.2 cm

H 6.8 cm

$x°$
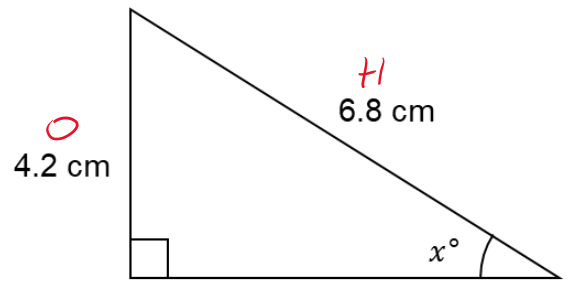
........38.1........°

(Total 3 marks)

4. Diagram NOT accurately drawn
Work out the value of y.
Give your answer correct to 3 significant figures.

$13 \div \sin 58$

$= 15.3293\ldots$

H y cm

58°

13 cm

O

........15.3........ cm
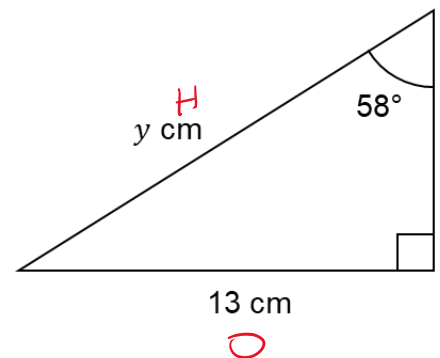
(Total 3 marks)

5. Diagram NOT accurately drawn
Work out the value of x.
Give your answer correct to 1 decimal place.

$\tan^{-1}\left(\dfrac{12}{17}\right)$

$= 32.2175\ldots$

O 12 mm

$x°$

17 mm A

........32.2........°

(Total 3 marks)

43
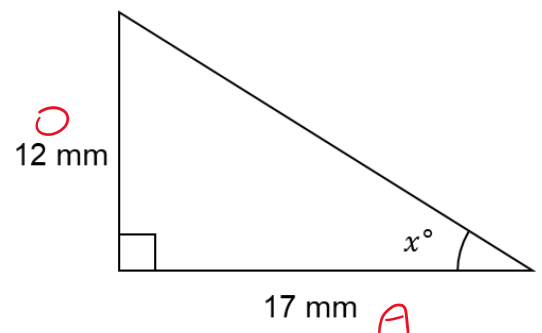
6. Diagram NOT accurately drawn
Work out the value of p.
Give your answer correct to 3 significant figures.

$\sqrt{5^2 + 12^2} = 13\ cm$

$13 \div \sin 63$

$= 14.5902...$

63°

p cm
H

13 cm

5 cm

12 cm
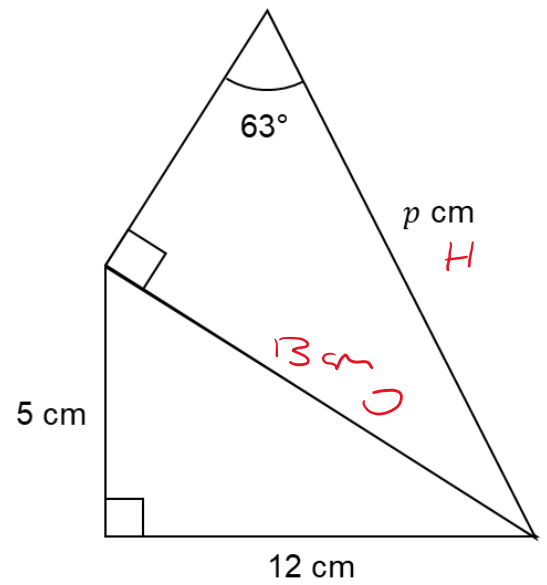
............14.6............... cm
(Total 5 marks)

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Exact values of Trigonometry

Things to remember:

Use these two triangles to derive relationships for sine, cosine and tangent.

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

You cannot use a calculator for these – leave your answers as exact numbers.
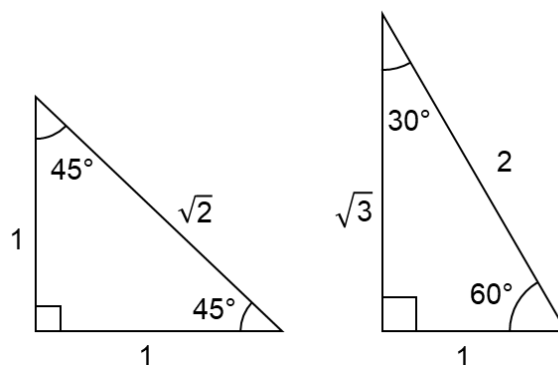
Questions:

1. Write down the exact value of cos (90°)

.... 0 ....
(Total 1 mark)

2. Write down the exact value of sin (45°)

.... $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ ....
(Total 1 mark)

3. Write down the exact value of cos (60°)

.... $\frac{1}{2}$ ....
(Total 1 mark)

4. Write down the exact value of tan (30°)

.... $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ ....
(Total 1 mark)

5. Write down the exact value of sin (30°)

.... $\frac{1}{2}$ ....
(Total 1 mark)

6. Write down the exact value of sin (90°)

.... 1 ....
(Total 1 mark)

7. Write down the exact value of cos (45°)

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

....
(Total 1 mark)

8. Write down the exact value of tan (60°)

$$\sqrt{3}$$

....
(Total 1 mark)

9. Write down the exact value of sin (0°)

$$0$$

....
(Total 1 mark)

10. Write down the exact value of cos (30°)

$$\frac{\sqrt{3}}{2}$$

....
(Total 1 mark)

11. Write down the exact value of tan (45°)

$$1$$

....
(Total 1 mark)

12. Write down the exact value of cos (0°)

$$1$$

....
(Total 1 mark)

13. Write down the exact value of sin (60°)

$$\frac{\sqrt{3}}{2}$$

....
(Total 1 mark)

Area and Perimeter Problem Solving

Things to remember:

- Start by deciding whether you need to calculate the area or the perimeter.
- You may have to split up the diagram into other shapes.
- Show your working clearly and keep units within your working out.

Questions:

1. Jessica is going to paint the wall shown in the diagram.
    There is a door in the wall.
    Jessica has enough paint to cover 12 m^2.
    Has she got enough paint?

    $(4 \times 2.5) - (2 \times 1)$

    $= 10 - 2$

    $= 8 \text{ m}^2$

    Yes she has enough paint.

    Diagram: rectangular wall 4 m wide, 2.5 m tall, with a door 1 m wide and 2 m tall.
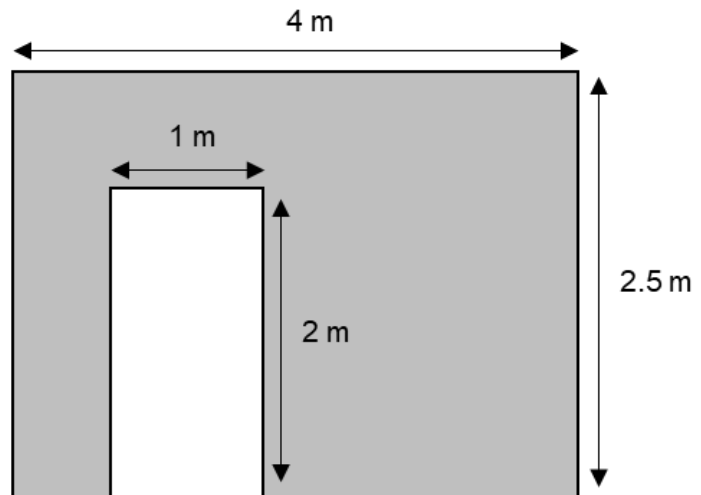
(Total 4 marks)

2. A farmer has a field to use for sheep a shown in the diagram.
    Each sheep will need 12 m^2.
    How many sheep can the farmer have in the field?

    $(11 \times 26) + (21 \times 12)$

    $= 286 + 252$

    $= 538 \text{ m}^2$

    $538 \div 12 = 44.833...$

    Diagram: L-shaped field with dimensions 11 m, 26 m, 21 m, 12 m, 32 m.

....................44....................

(Total 5 marks)

47
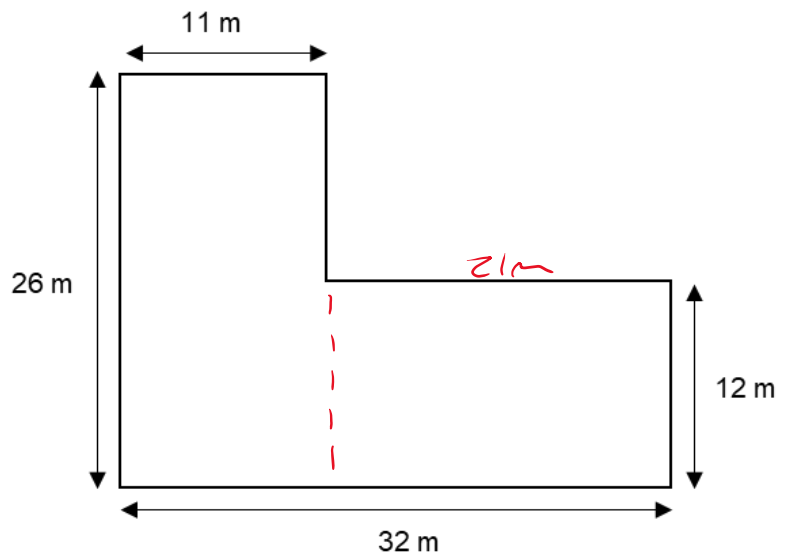
3. Aiden is going to paint the end of his house which is shown below.
One tin of paint will cover 25 m².
How many tins does Aiden need?

$(5 \times 6) + (\frac{1}{2} \times 5 \times 2)$

$= 30 + 5$

$= 35m^2$

2 m

6 m

5 m
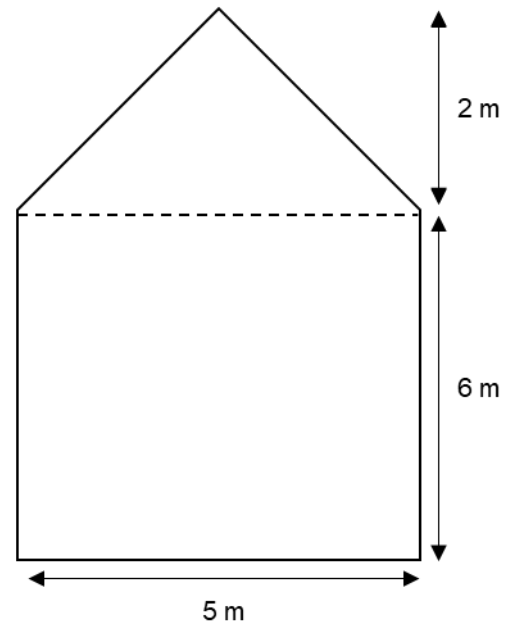
............2 tins............
(Total 4 marks)

4. Sarah's garden is shown in the diagram below.
Sarah needs a new fence.
Each fence panel is 1.5 m long and costs £30.
How much will Sarah's new fence cost?

House

3.2 m

2 m

1.5 m

5 m
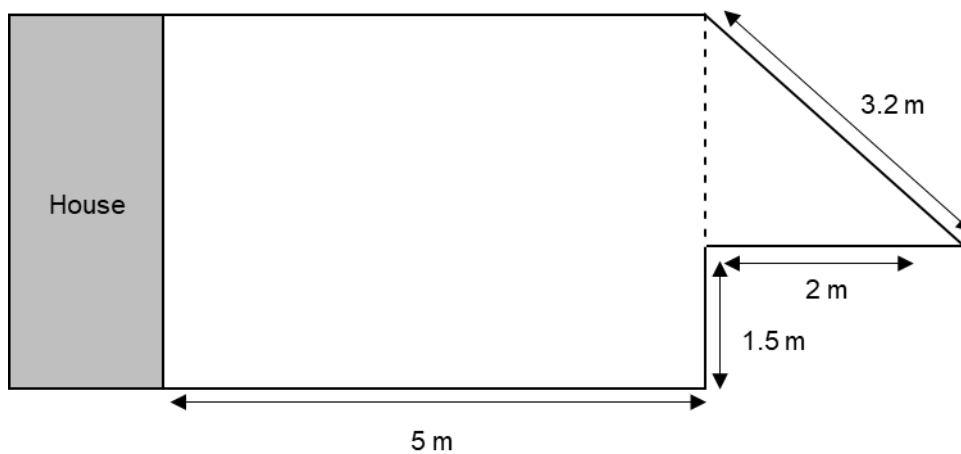
Perimeter: $5 + 1.5 + 2 + 3.2 + 5 = 16.7m$

$16.7 \div 1.5 = 11.133...$

12 panels × £30 = £360

£ ....360............
(Total 7 marks)

Contents ⌂

5. Lemar's garden is rectangular with a triangular pond in the middle, as shown in the diagram.
What percentage of Lemar's garden is pond?
Give your answer correct to 1 decimal place.
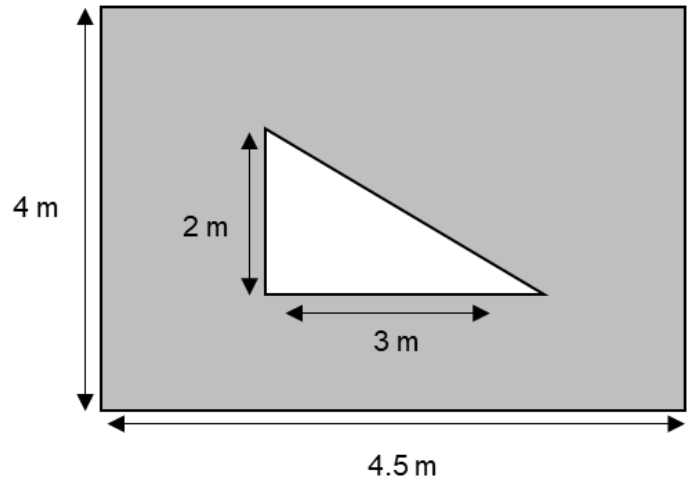
Pond: $\frac{1}{2} \times 2 \times 3 = 3m^2$

Garden: $4 \times 4.5 = 18m^2$

$\frac{3}{18} \times 100 = 16.66...$

4 m

2 m

3 m

4.5 m

..................16.7.................. %

(Total 4 marks)

6. Farmer Jane has a trapezium-shaped field as shown in the diagram.
She is going to sell the field for £28 per square metre.
How much will she receive for the sale of the field?
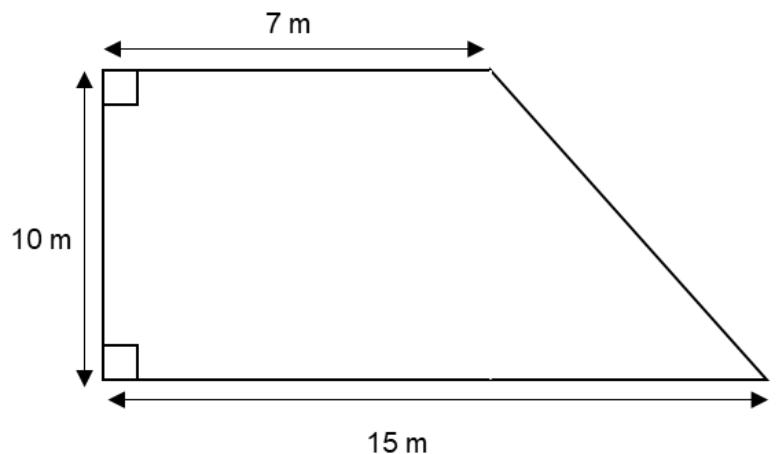
$\frac{1}{2}(7+15) \times 10$

$= 110m^2$

$110 \times 28 = £3080$

7 m

10 m

15 m

£ ....3080........
(Total 4 marks)

7. Jasminda is going to tile her bathroom wall.
49

The wall is 2.4 metres tall and 6 metres wide.
The tiles are 15 centimetres by 30 centimetres.
The tiles cost £12 for a pack of 30.
How much will it cost Jasminda to tile her bathroom wall?

16 tiles

2.4 m

20 tiles

6 m

15 cm

30 cm
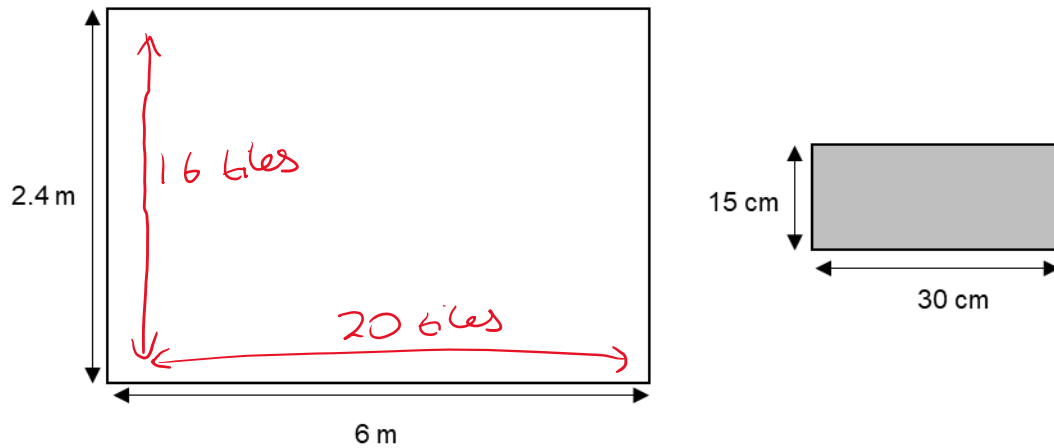
Total tiles: $16 \times 20 = 320$ tiles

$320 \div 30 = 10.66... \Rightarrow 11$ packs

$11 \times £12 = £132$

£ .... 132 ....
(Total 6 marks)

Area and Circumference of Circles

Things to remember:

- Area $= \pi r^2$
- Circumference $= \pi d$
- Remember the diameter is twice the radius.
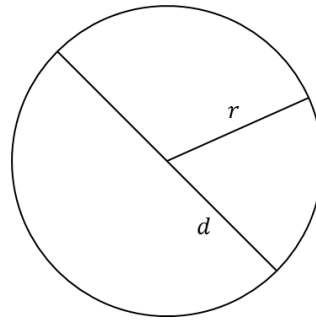
Questions:

1. The radius of this circle is 7 cm.
   Work out the area of the circle.
   Give your answer correct to 2 decimal places.

   $\pi \times 7^2 = 153.938...$
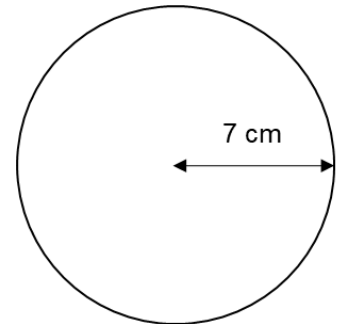
   ....153.94................. cm²
   (Total 2 marks)

2. The diameter of this circle is 3.8 cm.
   Work out the circumference of the circle.
   Give your answer correct to 3 significant figures.

   $3.8 \times \pi = 11.9380...$
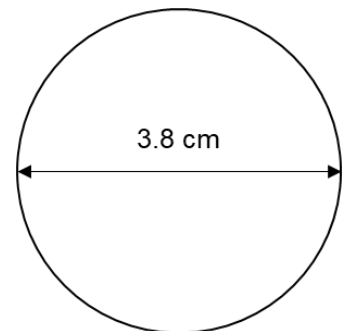
   ....11.9................. cm
   (Total 2 marks)

3. The diameter of this circle is 12 mm.
   Work out the area of the circle.
   Give your answer correct to 3 significant figures.

   $\pi \times 6^2 = 113.097...$
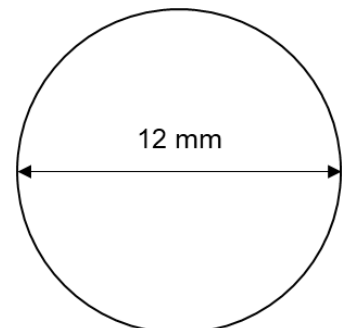
   ....113................. mm²
   (Total 2 marks)

4. The radius of this circle is 9.5 cm.
   Work out the circumference of the circle.
   Give your answer correct to 2 decimal places.

   $19 \times \pi = 59,6902\ldots$

   9.5 cm
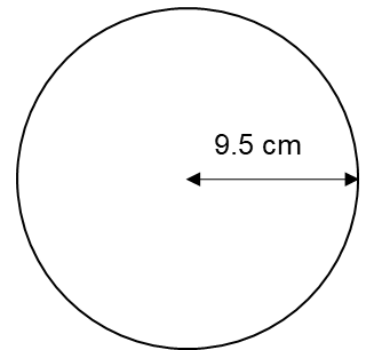
   ....59.69............... cm
   (Total 2 marks)

5. A circular field has a diameter of 28 metres.
   A farmer wants to build a fence around the edge of the field.
   Each metre of fence will cost £21.95
   Work out the total cost of the fence.

   $28 \times \pi = 87.7645\ldots \Rightarrow 88$ panels

   $88 \times £21.95 = 1931.6$

   £ ....1931.60.....................
   (Total 4 marks)

6. A semi-circle has an area of 30 cm^2
   Find the perimeter of the semi-circle.
   Give your answer correct to one decimal place.

   Radius: $\sqrt{\dfrac{30 \times 2}{\pi}} = 4.3701\ldots$

   Diameter: $8.7403\ldots$ (D)

   Perimeter: $\dfrac{\pi D}{2} + D = 22.409.-$
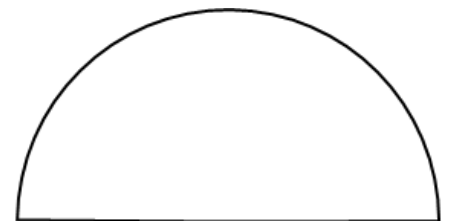
   ....22.5............... cm
   (Total 4 marks)

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7. A garden is formed by a square, $ABCD$, and a semi-circle.
BC is the diameter of the semi circle.
The radius of the semi-circle is 3 m.
The garden is going to be covered completely
with lawn seed.
A box of lawn seed covers 20 m².
How many boxes of lawn seed will be needed?
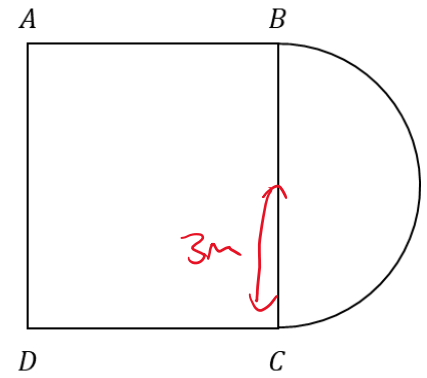
Square: $6^2 = 36m^2$

S. circle: $\dfrac{3^2 \times \pi}{2} = 14.137...m^2$

Total area: $50.137...m^2$

.... 3 boxes ....
(Total 5 marks)

8. The diagram shows a shaded ring formed by cutting a smaller circle
out of a larger circle.
The diameter of the smaller circle is 21 mm.
The diameter of the larger circle is 23 mm.
Find the area of the shaded ring.
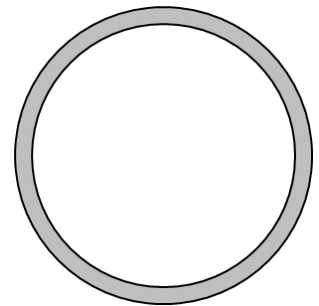
$\left(11.5^2 \times \pi\right) - \left(10.5^2 \times \pi\right)$

$= 69.115...$

.... 69.1 .... mm²
(Total 4 marks)

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Volume and Surface Area of Cones and Spheres

Things to remember:

- These formulae will be given in an exam if needed so you do not need to remember them!
- You might need to use Pythagoras' Theorem when calculating with cones
- Sometimes you will be asked to leave your answer in terms of π

Volume of cone $= \frac{1}{3}\pi r^2 h$
Curved surface area of cone $= \pi r l$

Volume of sphere $= \frac{4}{3}\pi r^3$
Surface area of sphere $= 4\pi r^2$
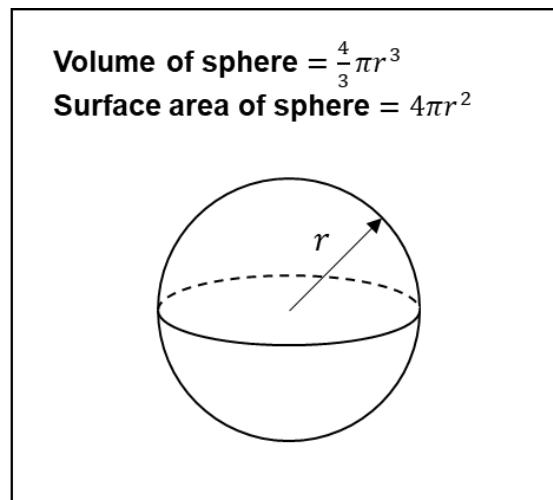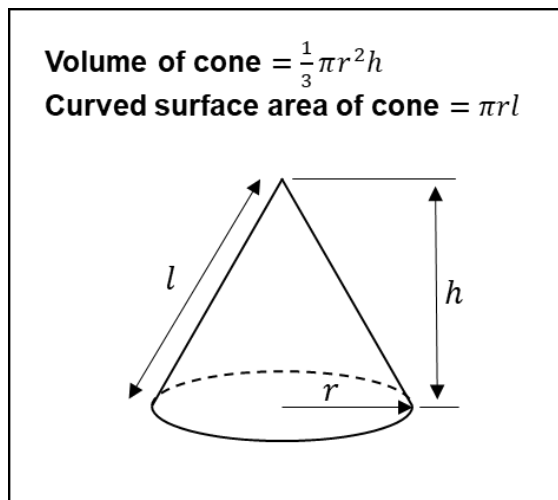
Questions:

1. The diagram represents a cone.
    The height of the cone is 8 cm.
    The diameter of the base of the cone is 6 cm.
    Calculate the **curved surface area** of the cone.
    Give your answer correct to 3 significant figures.

    $l = \sqrt{8^2 - 3^2} = 7.416\ldots$

    $\pi \times 3 \times 7.416\ldots = 69.896\ldots$

    8 cm

    6 cm
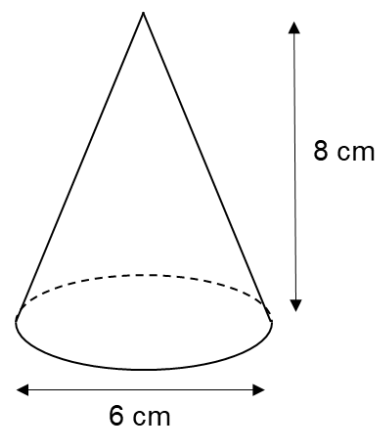
    ....69.9............... cm²
    (Total 4 marks)

2.   A sphere has a volume of 300 m³.
      Calculate the radius of the sphere.
      Give your answer correct to 3 significant figures.

$$\frac{4}{3}\pi r^3 = 300$$

$$r^3 = 71,619\ldots$$

$$r = 4.152\ldots$$

..................4.15.................... m
(Total 3 marks)

3.   The radius of the base of a cone is 3 cm.
      Its slant height is 6 cm.
      Calculate the volume of the cone.
      Give your answer correct to 3 significant figures.

Height : $\sqrt{6^2 - 3^2} = 5.196\ldots$ cm

$$\frac{1}{3} \times \pi \times 3^2 \times 5.196\ldots$$

$$= 48.9725\ldots$$

6 cm

3 cm
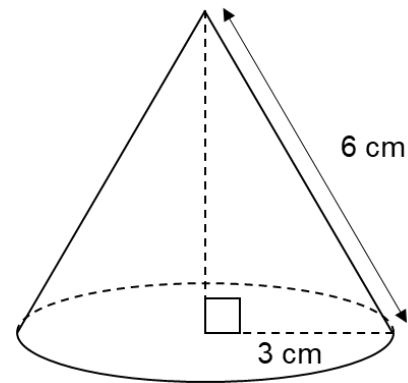
..................49.0.................... cm²
(Total 4 marks)

55

4. A clay bowl is in the shape of a hollow hemisphere.
The external radius of the bowl is 8 cm.
The internal radius of the bowl is 7 cm.
Calculate the volume of the clay bowl.

7 cm   8 cm
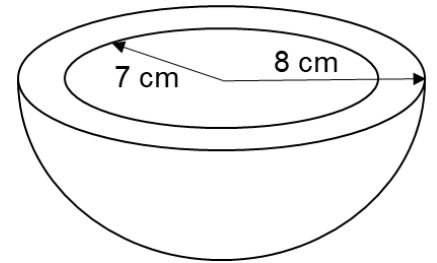
$$\frac{\left(\frac{4}{3} \times \pi \times 8^2\right) - \left(\frac{4}{3} \times \pi \times 7^3\right)}{2}$$

$$= \frac{268.082... - 205.250...}{2}$$

$$= 31.415...$$

.................................31.4.......................... cm³
(Total 4 marks)

5. The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.
The height of the cone is 13 cm.
The base of the cone has a diameter of 8 cm.
The diameter of the hemisphere is 8 cm.
Work out the total volume of the solid shape.
Give your answer in terms of π

13 cm

8 cm
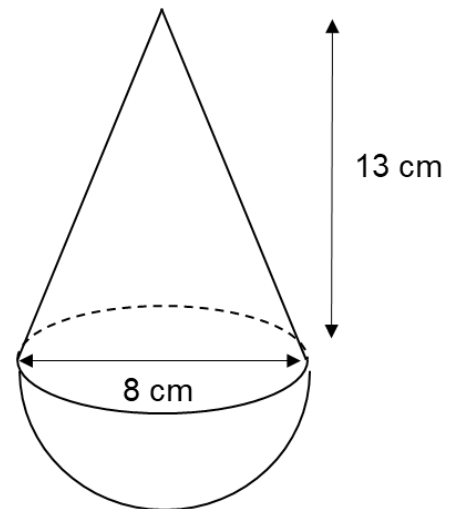
Cone: $\frac{1}{3} \times \pi \times 4^2 \times 13$

$$= \frac{208}{3} \pi$$

H. sphere: $\frac{2}{3} \times \pi \times 4^3$

$$= \frac{128}{3} \pi$$

$$\frac{208}{3} \pi + \frac{128}{3} \pi$$

...................112.11................. cm³
(Total 4 marks)

56

Compound Measures

Things to remember:

- Speed, density and pressure are examples of compound measures which means they are made up of two or more other measures
- You can use the units to derive the required formula if you forget – for example, a unit for speed is 'miles per hour', which implies distance divided by time
- You need to look out for units in these questions – examiners like to try to trick you out by requiring unit conversions
- You do not need to know the formula for pressure as you'll get given it if its needed

Questions:

1. A London airport is 200 miles from Manchester airport.
   A plane leaves Manchester airport at 6 am to fly to the London airport.
   The plane flies at an average speed of 120 mph.
   What time does the plane arrive at the London airport?

   $$\frac{200}{120} = \frac{5}{3} \text{ hour} = 1 \text{ hr } 40 \text{ mins}$$

   ....7:40 am..................
   (Total 4 marks)

2. A metal cone has a diameter of 4 cm and a height of 7 cm.
   The density of the metal is 7 g/cm^3
   Calculate the mass of the cone.

   Volume : $\frac{1}{3} \times \pi \times 2^2 \times 7$

   $= 29.321... \text{ cm}^3$

   Mass : $29.321... \times 7$

   $= 205.250...$

   7 cm

   4 cm
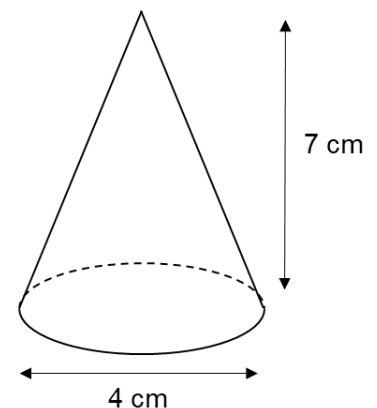
   ....205.25.......... grams
   (Total 4 marks)

Contents ⌂

3. Joe allows exactly one hour for his 40 mile journey. He leaves at 10 am.
He drives at 40 mph for the first 20 miles.
Unfortunately Joe gets stuck in traffic for 6 minutes.
At what speed will Joe need to average for the remainder of his journey to arrive at his destination at 11 am?

	1st	2nd	Total
S	40	50	
D	20	20	
T	0.5	0.4	1

6 Min delay!

.... 50 .... mph

(Total 4 marks)

4. Find the pressure exerted by a force of 1800 newtons on an area of 300 cm²
Give your answer in newtons/m²

$$pressure = \frac{force}{area}$$

$300cm^2 = 0.03m^2$

$Pressure = \frac{1800}{0.03}$

.... 60 000 .... newtons/m²

(Total 3 marks)

5. The mass of 3 m³ of tin is 21840 kg.
Work out the density of tin.
Include suitable units with your answer.

$\frac{21840}{3} = 7280$

.... 7280 kg/m³ ....

(Total 3 marks)

6. Material A has a density of 5 g/cm³

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Material B has a density of 6 g/cm³
20 g of Material A and 30 kg of Material B form Material C.
Work out the density of Material C.

	A	B	C
M	20	30	50
D	5	6	
V	4	5	9

50 ÷ 9 = 5.555...

............................5.56................ g/cm³
(Total 4 marks)

7.   The speed limit on a road is 40 mph.
A car drives 13 miles in 18 minutes.
Is the car breaking the speed limit?
You must show your workings.

18 mins = 0.3 hrs.

Speed: $\frac{13}{0.3}$ = 43.33...

Yes the car is breaking the speed limit.

(Total 3 marks)

59

Distance-Time Graphs

Things to remember:

- A horizontal line on a distance-time graph shows that the object is stationary (not moving), while a sloping line on a distance-time graph shows that the object is moving
- The speed of an object can be calculated from the gradient of a distance-time graph

Questions:

1.   Albert drove to the beach. On the way, he popped into his Grandma's house.
     The graph below shows part of his journey.
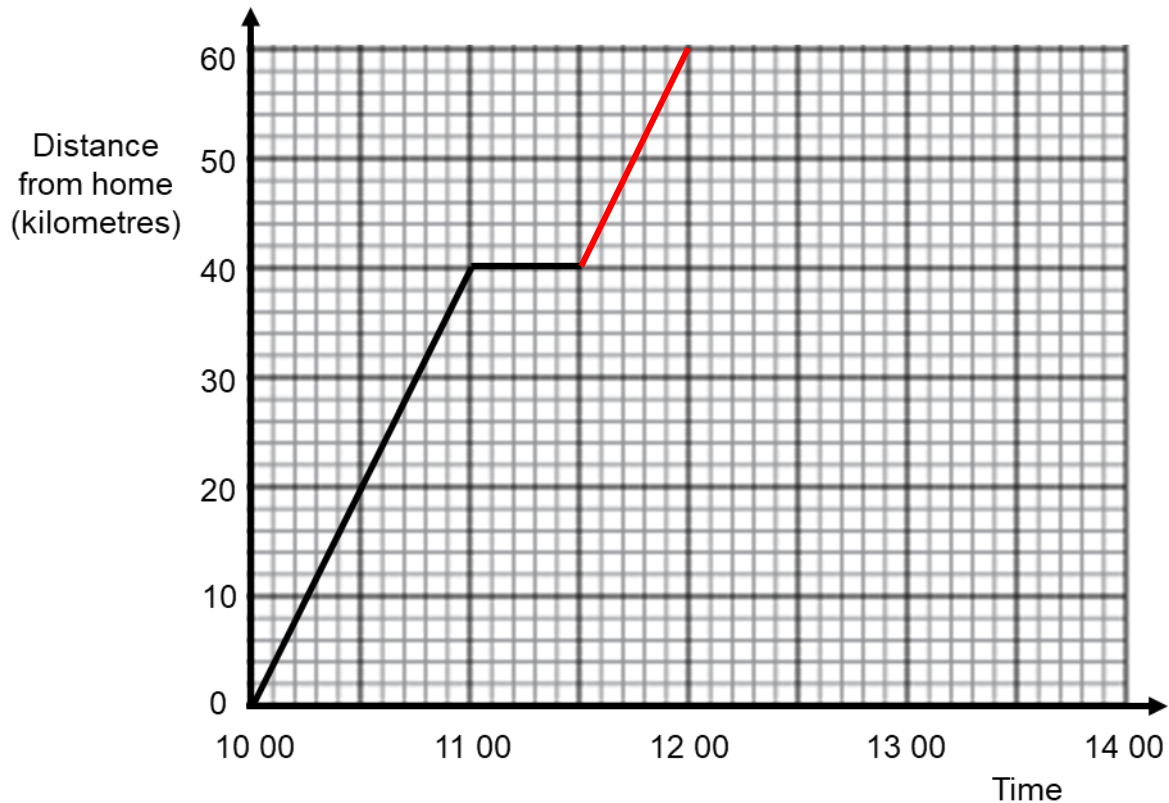
a)   How long was Albert at his Grandma's house for?

.... **30 mins** ....

(1)

b)   What was Albert's average speed for the first hour of his journey?

.... **40** .... km/h

(1)

Albert drove the last 20 miles in 30 minutes.

c)   Show this information on the graph.

(1)
(Total 3 marks)

2.   Six distance-time graphs are shown below.

Graph A   Graph B   Graph C

Graph D   Graph E   Graph F
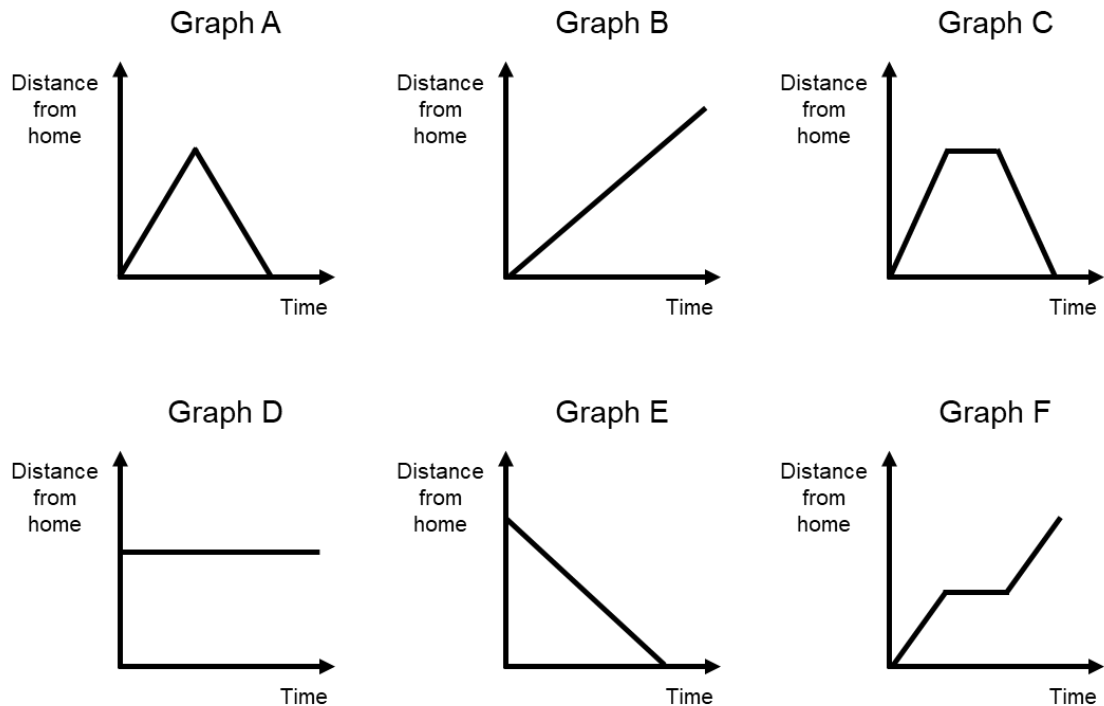
Each sentence in the table describes one of the graphs.
Write the letter of the correct graph next to each sentence.

Elsie leaves home and travels to school at a steady speed	B
Elsie travels to school and immediately returns home	A
Elsie leaves home and travels to school, stopping at the shop on the way	F
Elsie leaves school and travels home at a steady speed	E
Elsie stays at school	D
Elsie travels to school, stays there for a while, then returns home	C

(Total 3 marks)

3. Cecily drove to a family party and then back home.
   The party was at a restaurant that is 50 kilometres from her home.
   Cecily left home at 10:00 and arrived at the restaurant at 11:30.
   She stayed at the family party for 3 and a half hours.
   Cecily then drove home at a speed of 40 kilometres per hour.
   Show this information on the distance-time graph.
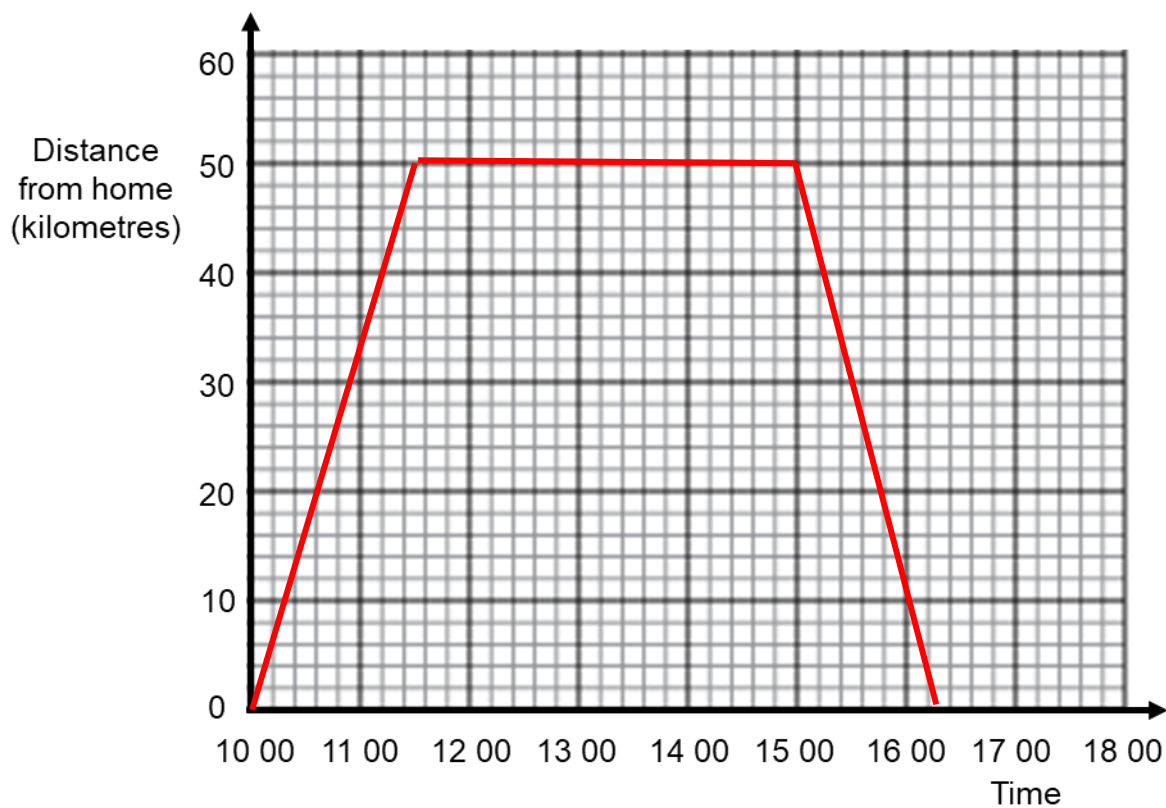
(Total 3 marks)

Averages from Frequency Tables

Things to remember:

- The modal value is the value that occurs most. From a table, this means the modal value is the one with the highest frequency
- The median value is the middle value when all items are in order. To find the value that is the median for a set of n items of data, add 1 to n and then divide by 2
- The mean is found by adding up all the numbers and dividing by how many numbers there are. To calculate this from a table, you need to divide the total number of items by the given frequency, ie. $\frac{\Sigma fx}{\Sigma f}$
- When given grouped data, you will have to estimate the mean by using the midpoint of each group in your calculations

Questions:

1. Shakir asked 24 of his friends how many pets they have.
   The results are shown in the table below.

Number of pets	Number of students
0	8
1	11
2	4
3	0
4	1

0
11
8
0
4

a) Write down the modal number of pets.

..................1.............................
(1)

b) Work out the median number of pets.

$$\frac{24+1}{2} = 12.5^{th}$$

.................1.........................
(2)

c) Calculate the mean number of pets.

$$\frac{11+8+4}{24} = 0.9583...$$

..................0.96.........................
(3)
(Total 6 marks)

Contents ⌂

2. Jesse did a survey of the number of cups of coffee some teachers in his school had drunk yesterday.
The frequency table shows his results.

Number of cups of coffee	Frequency	
2	1	2
3	3	9
4	5	20
5	8	40
6	5	30

a) Work out the number of teachers that Jesse asked

...................22...................
(1)

b) Work out the mean number of cups of coffee drunk.

$$\frac{2+9+20+40+30}{22} = 4.5909\ldots$$

...................4.59...................
(3)
(Total 4 marks)

3. Claire asked 40 colleagues how long their commute is to work.
The table shows some information about his results.

Time taken (m minutes)		Frequency	
$0 < m \le 10$	5	3	15
$10 < m \le 20$	15	8	120
$20 < m \le 30$	25	11	275
$30 < m \le 40$	35	9	315
$40 < m \le 50$	45	9	405

Work out an estimate for the mean time taken.

$$\frac{15+120+275+315+405}{40}$$

...................28.25...................
(Total 4 marks)

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4. Li Li owns a factory with 80 employees.
The table shows some information about the annual pay of these 80 employees.

Annual pay (£x)	Frequency	
10 000 < x ≤ 14 000 *12*	32	384 000
14 000 < x ≤ 16 000 *15*	24	360 000
16 000 < x ≤ 18 000 *17*	16	272 000
18 000 < x ≤ 20 000 *19*	6	114 000
20 000 < x ≤ 40 000 *30*	2	60 000

a) Write down the modal class interval.

$10000 < x ≤ 14000$

(1)

b) Find the class interval that contains the median.

$\frac{80+1}{2} = 40.5^{th}$

$14000 < x ≤ 16000$

(2)

c) Work out an estimate for the mean annual pay.

$$\frac{384\,000 + 360\,000 + 272\,000 + 114\,000 + 60\,000}{80}$$

£ 14 875

(3)

d) Why is your answer to part (c) an estimate?

Assumed each value is the midpoint of each group.

(1)
(Total 7 marks)

65

Scatter Graphs

Things to remember:

- Scatter graphs are a good way of displaying two sets of data to see if there is a relationship
- Check the scale carefully when plotting points. If it helps, write in more numbers on the scale
- Always draw a line of best fit to estimate values from your graph
- You can describe a relationship by using the words "positive correlation" or "negative correlation"
- It is important to remember that correlation does not imply causation

Questions:

1. An ice cream company records the number of ice creams it sells across 11 days and the temperature. The scatter graph shows this information.
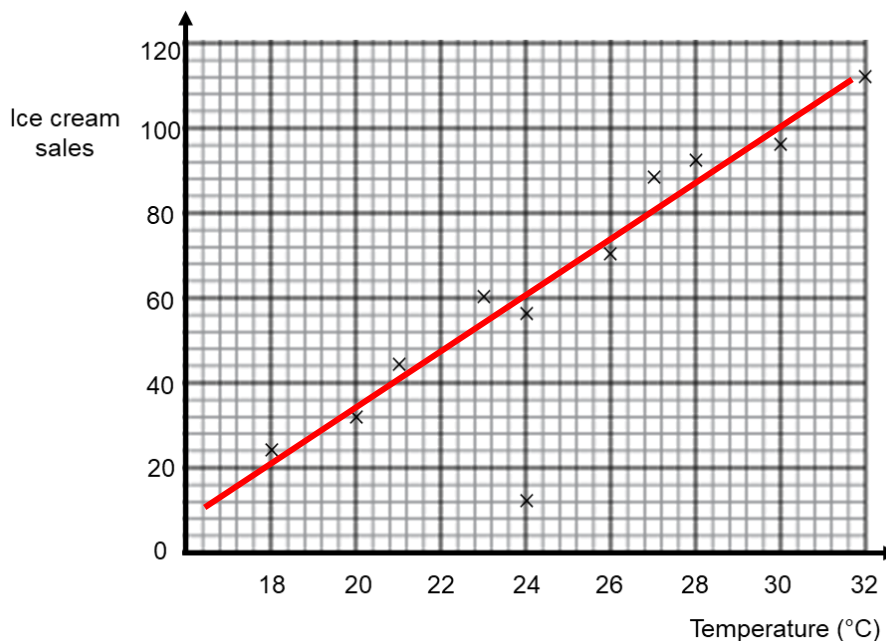
On day 12, the temperature was 25 °C and the company sold 72 ice creams.

a) Show this information on the scatter graph.

(1)

b) Describe the relationship between the temperature and the number of ice cream sales.

Positive correlation

(1)

One day, it rained.

c) What was the temperature on this day?

24 °C

(1)

d) On a day when the temperature is forecast to be 29 °C, how many ice cream sales can the company expect to make?

92

(2)

(Total 5 marks)

2. The table below shows the ages of 10 cars in a showroom and their prices.

Age (years)	8	3	7	2	6	3	4	2	6	9
Price (£1000s)	6	14	8	15	9	12	11	16	7	6
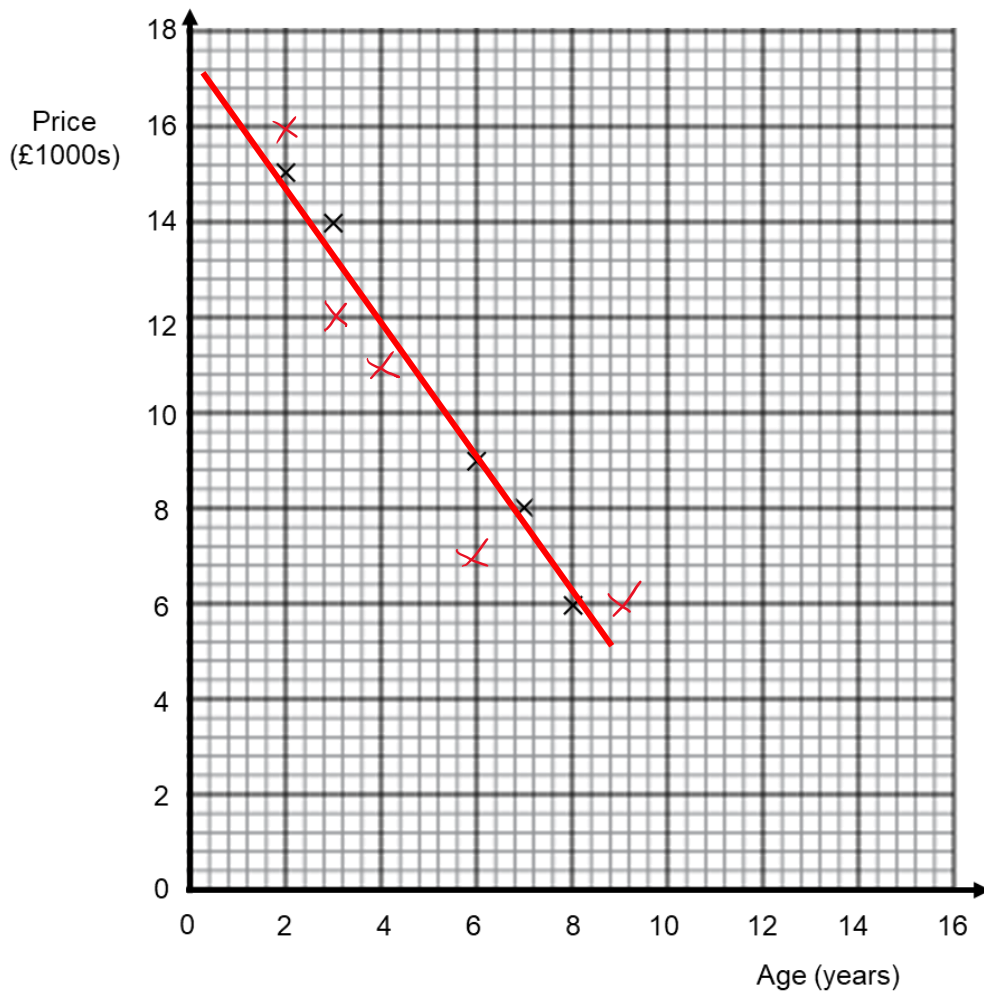
a) Complete the scatter graph to show the information in the table.
The first 5 points have been plotted for you.

(2)

b) Describe the relationship between the ages of the car and their prices.

Negative correlation

(1)

c) Estimate the price of a car that is 5 years old.

£ 10 500

(2)

d) The manager wants to use this data to estimate the price of a car that is 15 years old. Explain why this may not work.

Extrapolation – outside range of data.

(1)
(Total 6 marks)

67

3. The scatter graph shows the time spent revising and the test scores of ten students
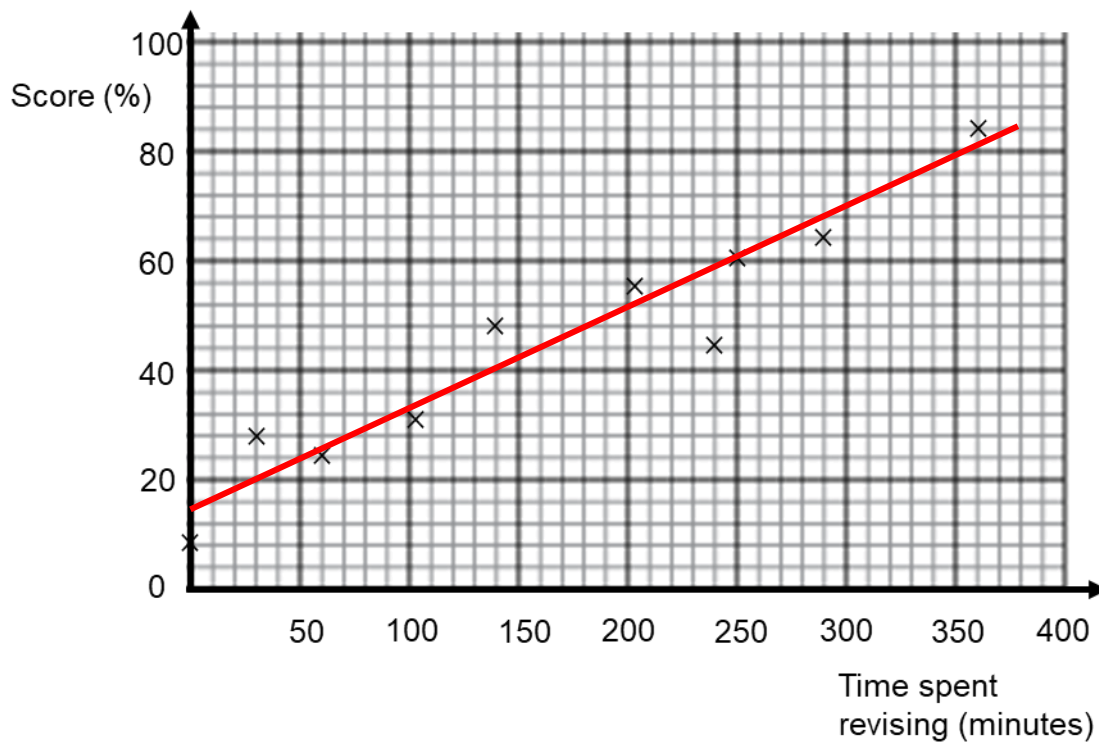
a) Describe the relationship shown in the scatter diagram.

...... Positive correlation ....

....

(1)

b) Draw a line of best fit on your scatter diagram.

(1)

c) Another student has spent 3 hours revising.
Use your line of best fit to estimate their test result.

.... 80 .... %

(1)

(Total 3 marks)

Time Series Graphs

Things to remember:

- Time series graphs show frequencies over time
- They are useful for identifying rising or falling trends in these frequencies
- Drawing a trend line can help to make estimations within the data range

Questions:

1. Olan is keeping track of how quickly his puppy is growing. He adopts the puppy at 8 weeks old and measures the mass of the puppy for 10 weeks.
    Olan's results are shown below.

Age (weeks)	7	8	9	10	11	12	13	14	15	16
Mass (grams)	1350	1430	1640	1760	1720	1750	1860	1920	1960	2010

a) Draw a time series graph below to show this information.

(3)
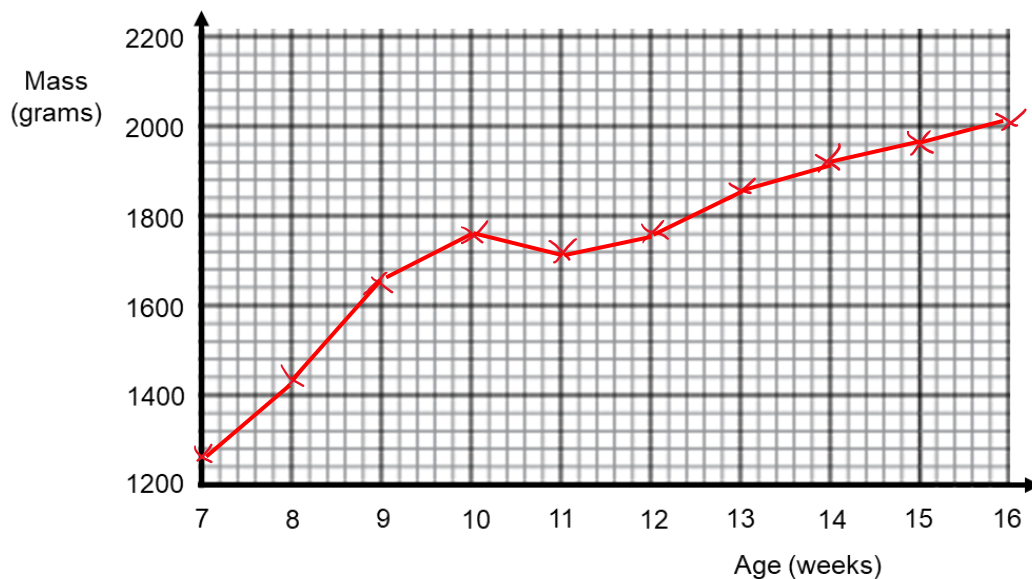
b) Between which two weeks was the biggest change?
    Explain how you can tell from your graph, rather than using the numbers in the table.

    Weeks 8-9 - steepest part of graph.

(2)

c) Olan's puppy was unwell during one of the weeks and did not eat as much food as usual.
    Suggest which week you think this was, giving a reason for your answer.

    Weeks 10-11 - dropped weight

(2)

(Total 7 marks)

2. This time series graph shows the information about the quarterly sales of ice creams in a shop.
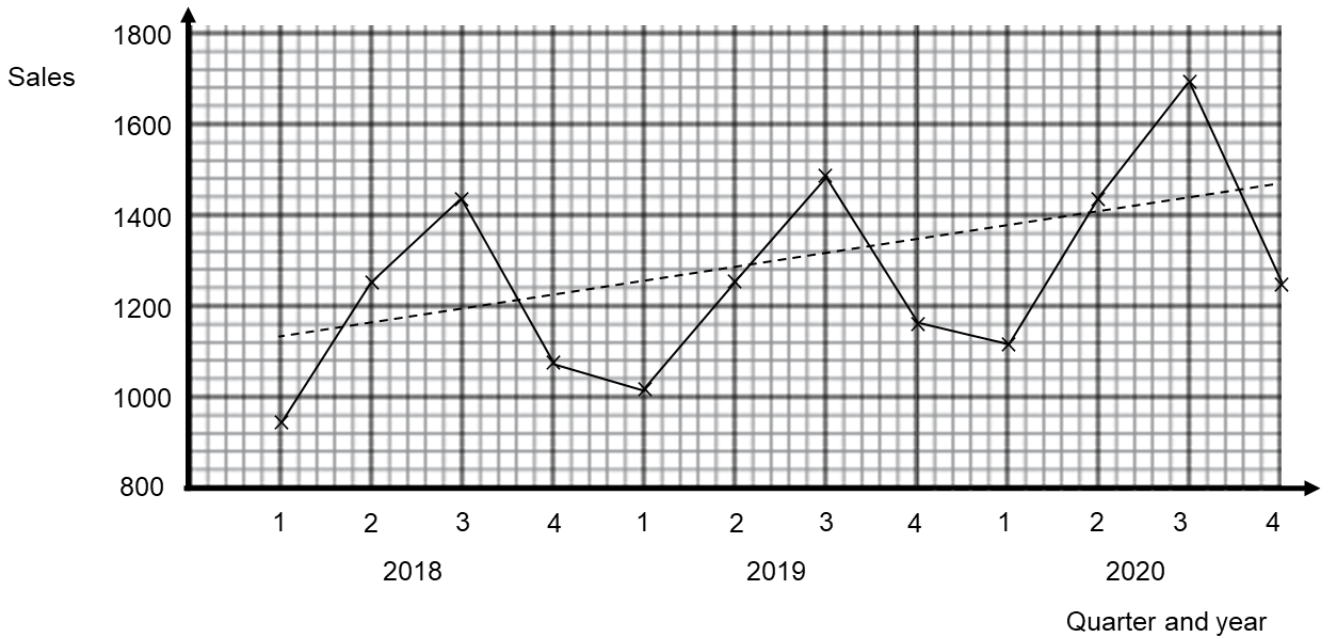
A trend line has been drawn on the graph.

a) Describe the trend in the quarterly sales of ice creams in the shop from 2018 to 2020.

........Rising trend...

....
(1)

b) i) Give an example of seasonal variation shown by the graph.

......Sales higher in 2nd/3rd quarters.......

....

ii) Suggest a reason for this seasonal variation.

......More ice creams are sold in warmer

.....weather...
(2)
(Total 3 marks)

3. Casey found the following information about the average price of a cinema ticket in the UK.

Year	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016
Average price (£)	5.05	5.20	5.44	5.95	6.06		6.53	6.72	7.21	7.41

(Source: UK Cinema Association)

He did not find the average price for 2012.

Casey's first six average prices have been plotted on the grid below.

a) Plot the average price for each of 2014, 2015 and 2016.

(2)
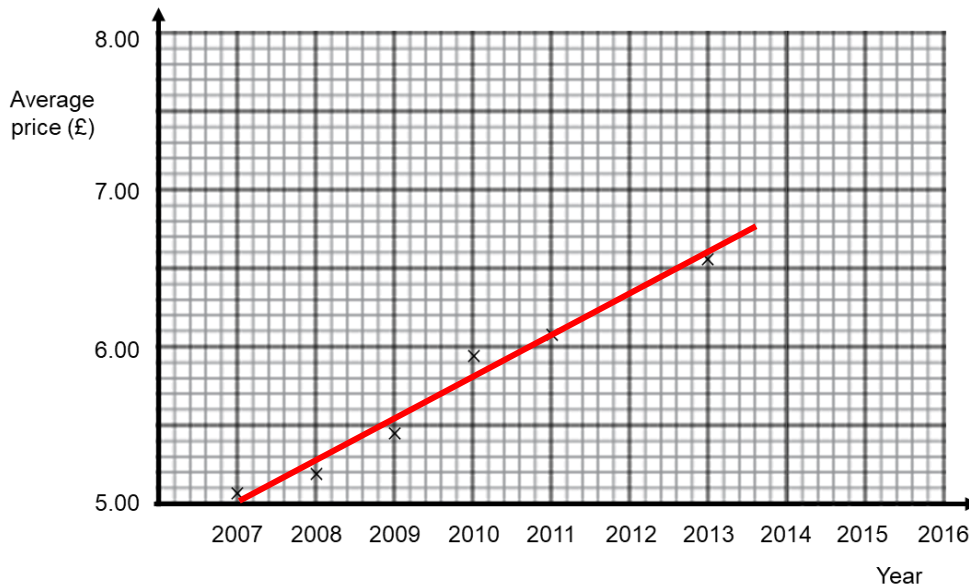
b) i) On the grid, draw a trend line for Casey's data.

(1)

ii) Describe the trend in the average price of cinema tickets in the UK from 2007 to 2016.

Rising trend

(2)

Rhian wants to use a trend line for Casey's data in order to find an estimate of the average price of a cinema ticket in 2012 **and** 2020.

c) Explain whether each of these is a sensible thing to do.
You do not need to work out the estimates.

2012 - interpolation - will give reliable result.

2020 - interpolation - unreliable result.

(3)

(Total 8 marks)

Stratified Sampling

Things to remember:

- Random sampling is where every member of the population has an equal chance of being chosen, which makes it fair.
- Stratified sampling is the best way to reflect the proportions of the population accurately
- Stratified sample = $\frac{\text{total in group}}{\text{total in population}}$ × sample size

Questions:

1. A school has 1300 students.
   Each student studies one of Mandarin or Spanish or German or French.
   The table shows the number of students who study each of these languages.

Language	Number of students
Mandarin	290
Spanish	242
German	396
French	372

   An inspector wants to look at the work of a stratified sample of 50 of these students.
   Find the number of students studying each of these languages that should be in the sample.

   Mandarin: $\frac{50}{1300} \times 290 = 11.1538... \approx 11$

   Spanish: $\frac{50}{1300} \times 242 = 9.3076... \approx 9$

   German: $\frac{50}{1300} \times 396 = 15.2307 \approx 15$

   French: $\frac{50}{1300} \times 372 = 14.3076... \approx 14$

   Round up one of these

   Mandarin ....11...........................

   Spanish ....9 or 10.................

   German ....15.......................

   French ....14 or 15..............

   (Total 3 marks)

2.   The grouped frequency table shows information about the weights, in kilograms, of 20 students, chosen at random from Year 10.

Weight (w kg)	Frequency
$50 \leq w < 60$	5
$60 \leq w < 70$	8
$70 \leq w < 80$	6
$80 \leq w < 90$	1

There are 200 students in Year 10.
Work out an estimate for the number of students in Year 10 whose weight is between 60 kg and 70 kg.

$200 \div 20 = 10$

$10 \times 8 = 80$

.... 80 ....
(Total 2 marks)

3.   a)   Explain what is meant by

      i)   a random sample,

.... Every member of the population has an ....
.... equal chance of selection ....

      ii)   a stratified sample.

.... The sample is representative of the proportions ....
.... within the population ....
(2)

      b)   There are 1250 students in Fairyhill High School.
            Aurora takes a random sample of 100 students.
            She asks these 100 students which subject they like best.
            They can choose English or maths or science.
            Aurora is going to use her results to work out an estimate of how many of the 1250 students like maths best.
            Explain how.

.... Assuming that the sample is representative of ....
.... the population, she can multiply her answers ....
.... by 12.5 to estimate preference for the population. ....
....
(2)
(Total 4 marks)

4.   The table shows some information about the pupils at Easter School.

Year group	Boys	Girls	Total
Year 7	96	108	204
Year 8	84	93	177
Year 9	79	88	167
Total	259	289	548

Andi carries out a survey of the pupils at Easter School.
She takes a sample of 60 pupils, stratified by both year group and gender.

a)   Work out the number of Year 9 boys in her sample.

$$\frac{60}{548} \times 79 = 8.6496...$$

...................................9...................................
(2)

b)   Describe a method that Andi could use to take a random sample of Year 9 boys.

Allocate each student a number, use a random number generator to select 60 numbers, select corresponding year 9 boys.

(2)
(Total 4 marks)

Probability Trees

Things to remember:

- The branches must sum to 1
- Read the question carefully to decide if it is with replacement or without replacement
- 'And' means × and 'or' means +

Questions:

1. Rhys has two bags of counters.
   The first bag contains 3 red counters and 2 blue counters.
   The second bag contains 2 red counters and 5 blue counters.
   Rhys takes one counter at random from each bag.

   a) Complete the probability tree diagram.

   First bag　　　　　　　**Second bag**

   $\frac{3}{5}$　Red

   $\frac{2}{7}$　Red

   $\frac{5}{7}$　Blue

   $\frac{2}{5}$　Blue

   $\frac{2}{7}$　Red

   $\frac{5}{7}$　Blue

   (2)
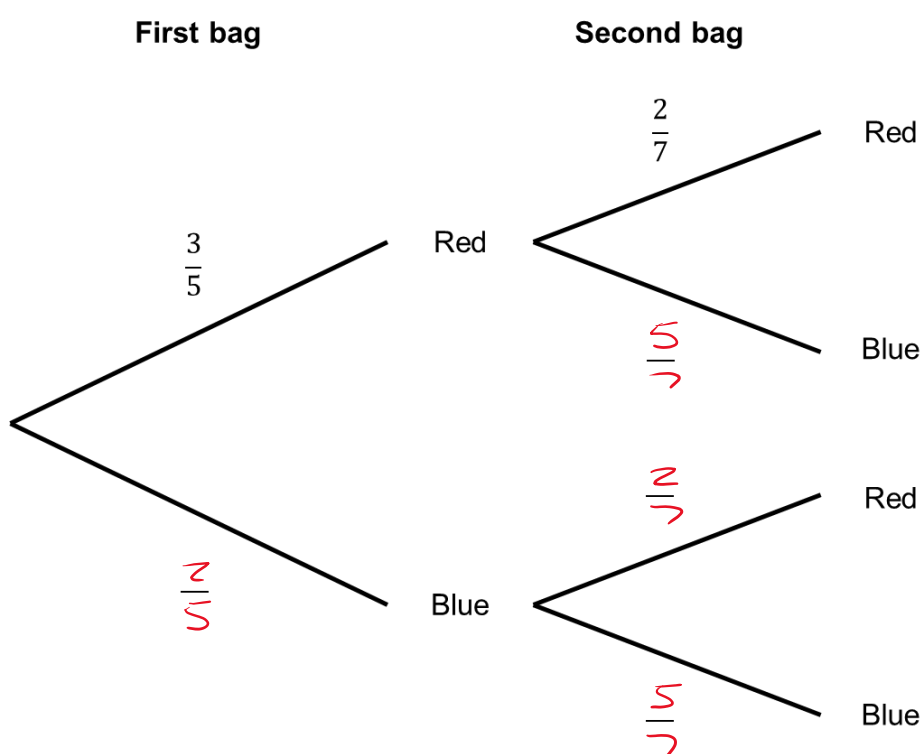
   b) Work out the probability that Rhys takes two red counters.

   $$P(\text{red and red}) = \frac{3}{5} \times \frac{3}{5} = \frac{9}{25}$$

   $$\frac{9}{25}$$

   ....

   (2)
   (Total 4 marks)

2. Hannah is going to play one game of tennis and one game of badminton.
The probability she will win the game of tennis is 0.6
The probability she will win the game of badminton is 0.5

a) Complete the probability tree diagram.

Tennis   **Badminton**

                                                    Hannah
                                            0.5   wins
                        Hannah
                        wins
        0.6
                                            .0.5.   Hannah
                                                    loses

                                            0.5   Hannah
                                                    wins
        0.4
                        Hannah
                        loses
                                            0.5   Hannah
                                                    loses

(2)
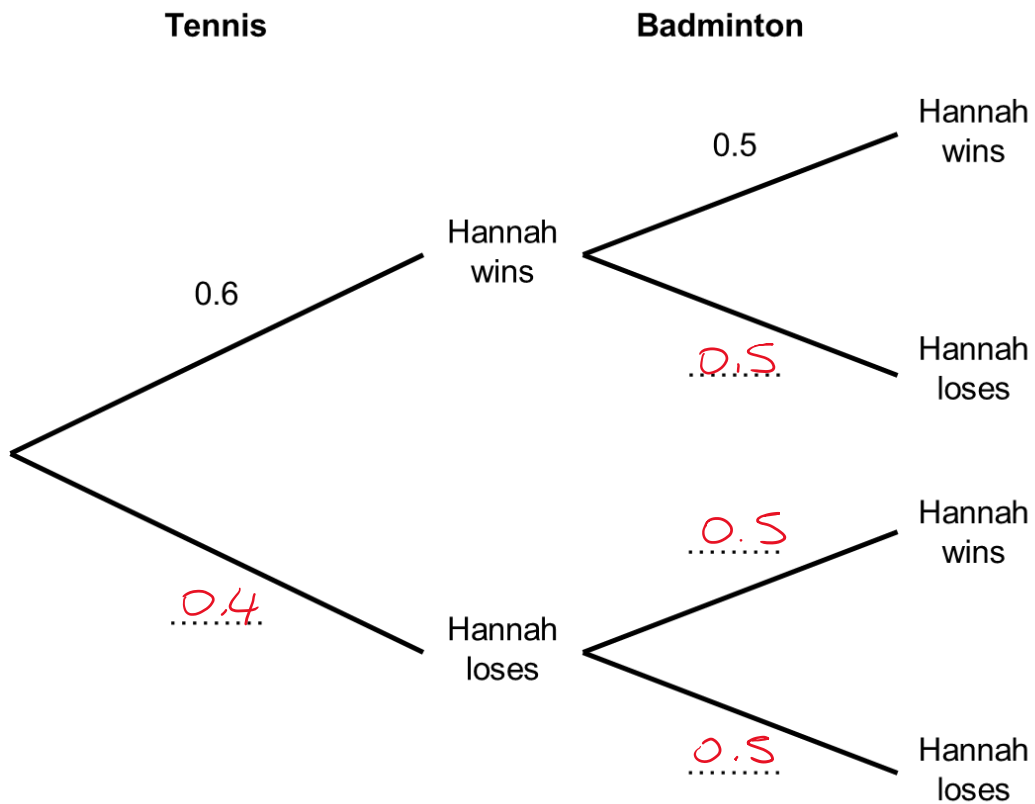
b) Work out the probability that Hannah will win both games.

P(win and win) = 0.6 × 0.5 = 0.3

..................0.3..........................

(2)
(Total 4 marks)

3. Matthew puts 4 red counters and 5 blue counters in a bag.
    He takes at random a counter from the bag.
    He writes down the colour of the counter.
    He puts the counter in the bag again.
    He then takes at random a second counter from the bag.

    a) Complete the probability tree diagram.

    First counter   **Second counter**

$$\frac{4}{9}$$ Red

$$\frac{4}{9}$$ Red

$$\frac{5}{9}$$ Blue

$$\frac{5}{9}$$ Blue

$$\frac{4}{9}$$ Red

$$\frac{5}{9}$$ Blue

$$\frac{5}{9}$$ Blue

(2)
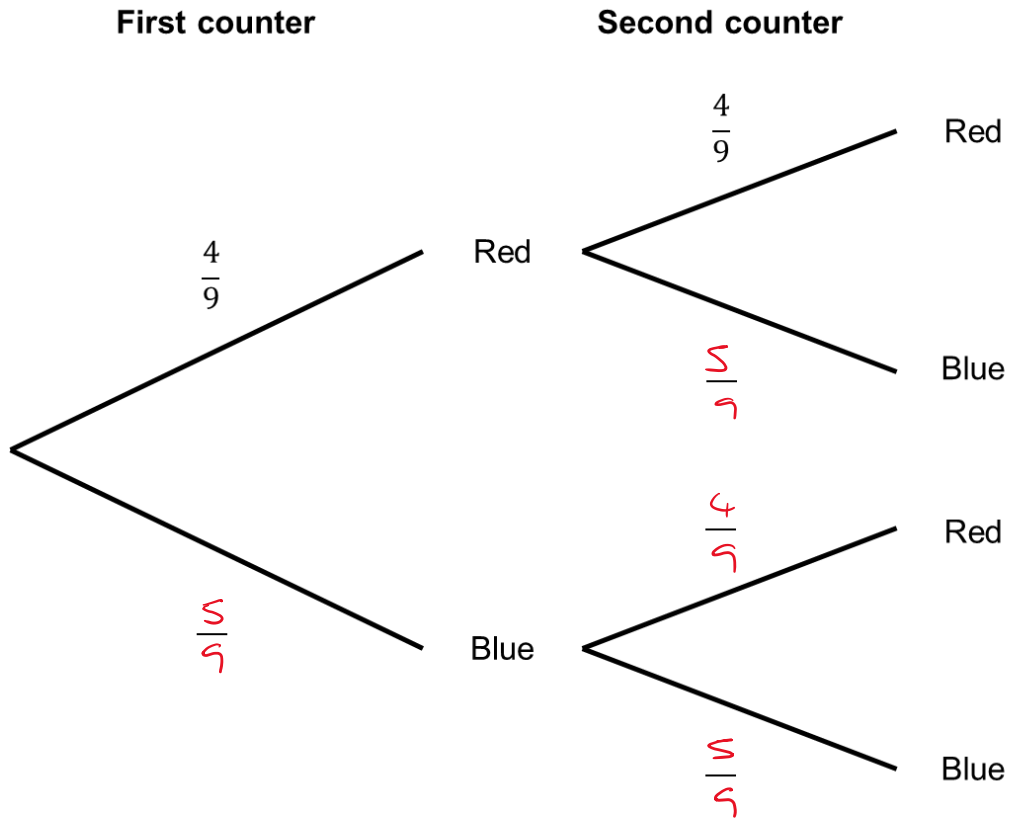
    b) Work out the probability that Matthew takes a counter of each colour.

$$P(\text{red and blue or blue and red})$$

$$= \frac{4}{9} \times \frac{5}{9} + \frac{5}{9} \times \frac{4}{9}$$

$$= \frac{20}{81} + \frac{20}{81}$$

$$\frac{40}{81}$$

....

(2)
(Total 4 marks)

77

4. Bart plays a game where he can win, draw or lose.
The probability Bart wins any game 0.5.
The probability Bart draws any game is 0.3
Bart plays two games.

a) Complete the probability tree diagram.

First game   **Second game**

0.5 — Win
0.3 — Draw
0.2 — Lose
(Win branch, 0.5)

0.5 — Win
0.3 — Draw
0.2 — Lose
(Draw branch, 0.3)

0.5 — Win
0.3 — Draw
0.2 — Lose
(Lose branch, 0.2)

(2)
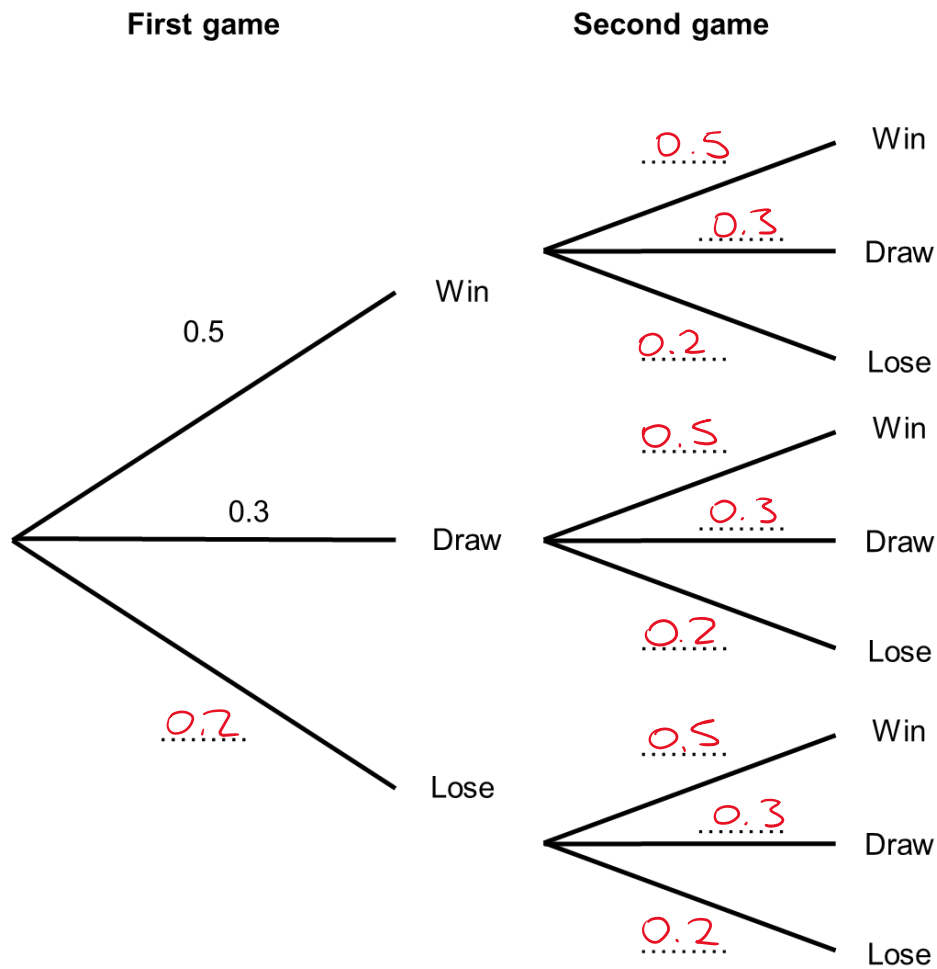
b) Work out the probability that Bart wins both games.

P(win and win) = 0.5 × 0.5 = 0.25

0.25

(2)
(Total 4 marks)

Venn Diagrams

Things to remember:

- Venn diagrams are used to sort and organise data
- Make sure you sort the data into the correct part of the Venn diagram – pay attention any overlaps!
- Don't forget to include the everything set (the box around the outside)

Questions:

1. $\varepsilon = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$
   A = multiples of 2
   B = multiples of 3

   Complete the Venn diagram
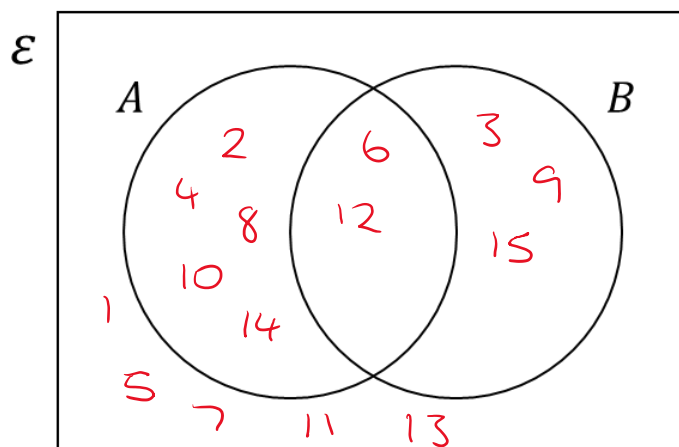
(Total 3 marks)

2.	There are 200 students in year 11.
	23 students study sport and geography.
	41 students only study sport.
	61 students do not study sport or geography.

	a)	Complete the Venn diagram
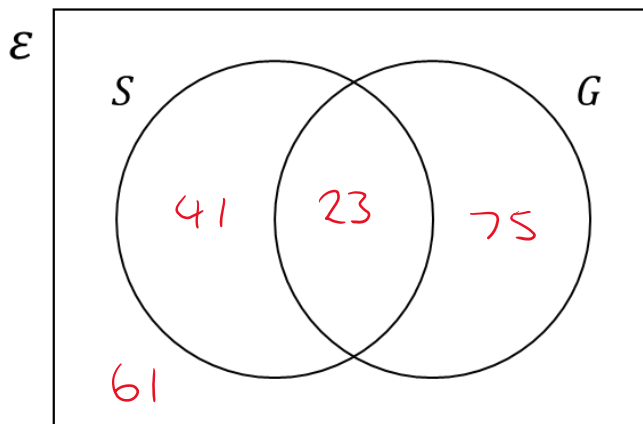
(3)

	b)	Work out how many students study only geography.

.... 75 ....

(1)

(Total 4 marks)

3.	In a company there are 110 workers.
	90 workers like tea.
	41 workers like coffee.
	25 workers like both tea and coffee.
	Work out how many workers like neither tea or coffee.
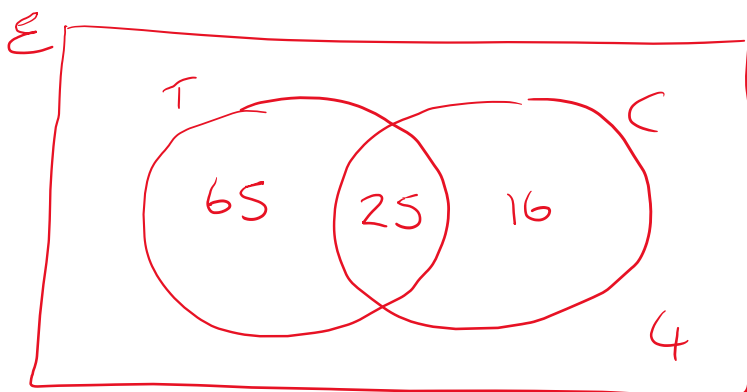
.... 4 ....

(Total 3 marks)

4. Libby asked 80 people which sports they enjoy from football, cricket and rugby.
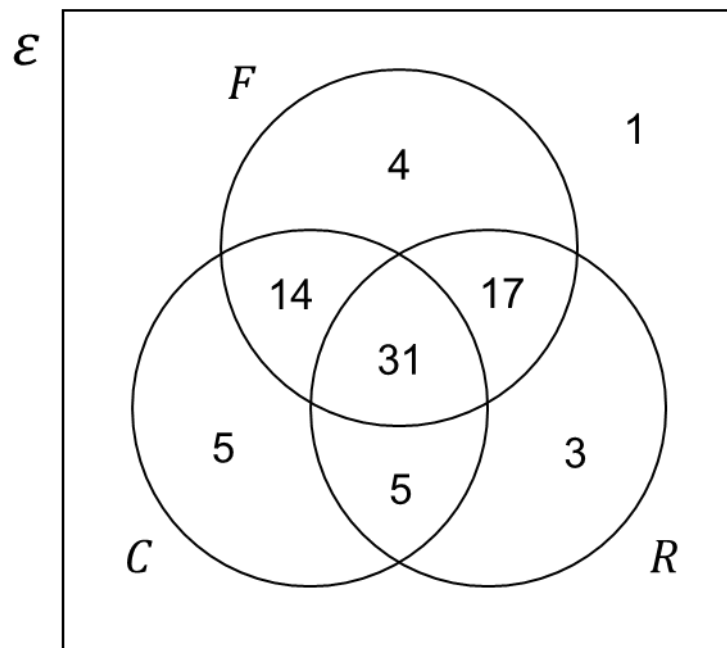
a) How many people enjoy all three sports?

....................................31........................

(1)

b) How many people enjoy football and cricket but not rugby?

..............................14.............................

(1)

c) How many people enjoy football and rugby but not cricket?

...........................17.................

(1)

d) Work out which sport is enjoyed by the most number of people.

Football : 66
Cricket : 55
Rugby : 56

...................Football............................

(3)
(Total 6 marks)

Direct and Inverse Proportion

Things to remember:

- A **directly proportional** relationship is where one variable **increases** at the same rate as another variable
- An **inversely proportional** relationship is where one variable **decreases** at the same rate that another variable increases

Questions:

1.   8 pens cost £18.40
      Work out the cost of 5 pens.

      £18.40 ÷ 8 = £2.30

      £2.30 × 5 = £11.50

      £ .... 11.50 ....

      (Total 2 marks)

2.   A machine fills 160 jars in 4 hours.
      How many jars can it fill in 7 hours?

      160 ÷ 4 = 40

      40 × 7 = 280

      .... 280 ....

      (Total 2 marks)

3.   3 cakes and 4 scones cost £6.95
      5 cakes cost £4.25
      Work out the cost of one scone.

      £4.25 ÷ 5 = 85p

      3 × 85p = £2.55

      $\frac{£6.95 - £2.55}{4}$ = £1.10

      £ .... 1.10 ....

      (Total 2 marks)

4. There are 500 sheets in a pack of paper. 500 sheets of paper weigh 2.5 kg
    Work out the weight of 200 sheets of paper.

    $2.5kg \div 500 = 0.005 kg$

    $0.005 kg \times 200 = 1 kg$

    ....1................... kg
    (Total 2 marks)

5. It takes 3 painters 5 days to complete a job.
    How long would it take one painter?

    $3 \times 5 = 15$ days

    ....15 days...........
    (Total 2 marks)

6. It takes 3 machines 6 days to complete a job.
    Work out how long it would take 2 machines to complete it.

    $3 \times 6 = 18$

    $18 \div 2 = 9$

    ....9 days...........
    (Total 2 marks)

7. It takes 4 bricklayers 9 days to complete a job.
    How long would it take 6 of them to complete the job?

    $4 \times 9 = 36$

    $36 \div 6 = 6$

    ....6 days...........
    (Total 2 marks)

8.   Here are four graphs.

A

B

C

D
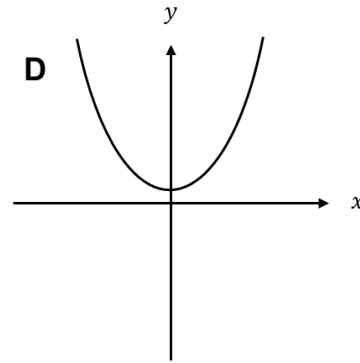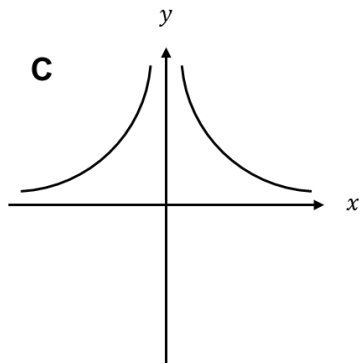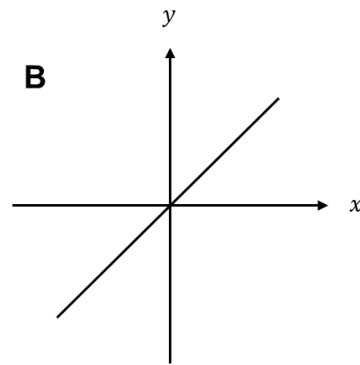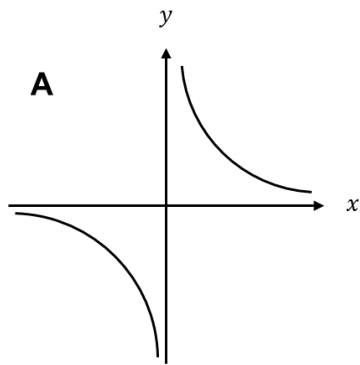
Match each graph with a statement in the table below.

Proportionality relationship	Graph letter
y is directly proportional to x	B
y is inversely proportional to x	A
y is directly proportional to x^2	D
y is inversely proportional to x^2	C

(Total 2 marks)

Comparing fractions, decimals and percentages

Things to remember:

$\frac{?}{100}$ and simplify

Percentage → Fraction

× 100

Divide
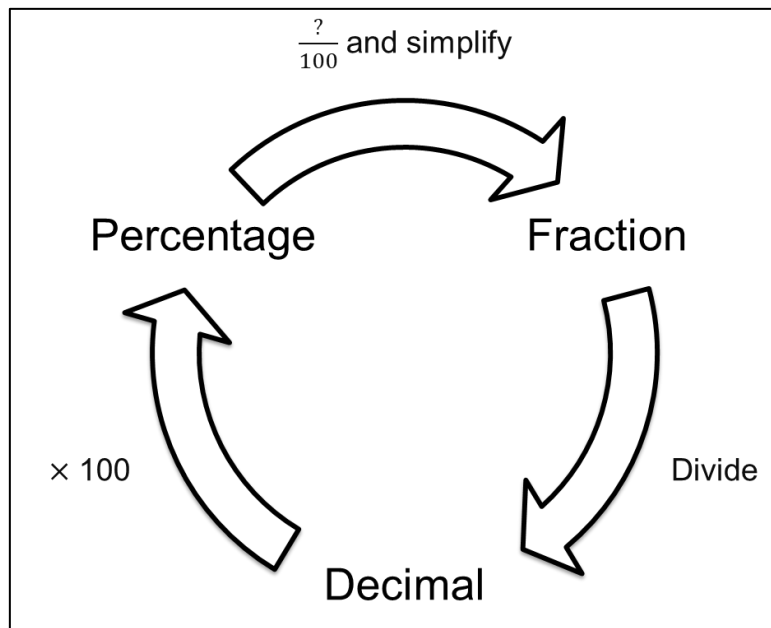
Decimal

Questions:

1. a) Write $\frac{7}{10}$ as a decimal.

    0.7

    (1)

    b) Write 0.45 as a percentage.

    45%

    (1)

    c) Write 30% as a fraction.
        Give your fraction in its simplest form.

    $\frac{30}{100}$

    $\frac{3}{10}$

    (2)
    (Total 4 marks)

2. a) Write 0.7 as a fraction.

    $\frac{7}{10}$

    (1)

    b) Write 0.3 as a percentage.

    30%

    (1)

    c) Write $\frac{8}{12}$ in its simplest form.

    $\frac{2}{3}$

    (1)
    (Total 3 marks)

3.   Write these numbers in order of size. Start with the smallest number.

75% $\frac{7}{8}$ 0.25 $\frac{1}{2}$ $\frac{2}{3}$

0.75   0.875   0.25   0.5   0.666...

...........$0.25, \frac{1}{2}, \frac{2}{3}, 75\%, \frac{7}{8}$...
(Total 2 marks)

4.   Write these numbers in order of size. Start with the smallest number.

0.6 $\frac{2}{3}$ 65%   0.606

0.6   0.666...   0.65   0.606

...........$0.6, 0.606, 65\%, \frac{2}{3}$...
(Total 2 marks)

5.   Amelia, Bronwyn and Catherine completed a test.
The total for the test was 75 marks.
Amelia got 56% of the 75 marks.
Bronwyn got $\frac{8}{15}$ of the 75 marks.
Catherine got 43 out of 75
Who got the highest mark?
You must show all your working.

56% of 75 = 0.56 × 75 = 42

$\frac{8}{15}$ of 75 = 40

......Catherine.........................
(Total 3 marks)

6. Martha had collected £640 in a sponsored event.
   She gave $\frac{1}{2}$ of the amount collected to her local youth club.
   She gave 30% of the amount collected to a children's hospital.
   She gave the rest of the money to a mountain rescue group.

   a)   How much money did Martha give to the mountain rescue group?

   $\frac{1}{2}$ of £640 = £320

   30% of £640 = £192

   £640 − (£320 + £192) = £128

   £ .... 128 ....

   (3)

   b)   What percentage of the £640 did Martha give to the mountain rescue group?

   $\frac{128}{640} \times 100$

   .... 20 .... %

   (2)

   (Total 5 marks)

7. There are 500 passengers on a train.
   $\frac{7}{20}$ of the passengers are men.
   40% of the passengers are women.
   The rest of the passengers are children.
   Work out the number of children on the train.

   $\frac{7}{20}$ of 500 = 175

   40% of 500 = 200

   500 − (175 + 200) = 125

   .... 125 ....

   (Total 3 marks)

Contents ⌂

Compound Interest and Depreciation

Things to remember:
- New amount = original amount × multiplier^n

Number of years

Questions:

1. Yasmin invests £400 for two years at 5% compound interest, paid yearly.
   Amin says that the interest Yasmin will receive will be £40.
   Is Amin right?
   Explain your answer.

   $400 \times 1.05^2 = £441$

   No – Amin has used simple interest, not compound interest

   (Total 3 marks)

2. San invests £500 at a compound interest rate of $R\%$ per annum.
   The value, £V, of this investment after n years is given by the formula

   $$V = 500 \times (1.045)^n$$

   Write down the value of R.

   4.5

   (Total 1 mark)

3. Gerrie invests £2000 for 3 years in a savings account.
   She was paid 4% per annum compound interest.
   Work out how much was in her account at the end of the 3 years.

   $2000 \times 1.04^3 = 224.9728$

   £ 224.97

   (Total 3 marks)

4. The value of a car depreciates by 15% each year.
   At the end of 2019 the value of the car was £8460
   Work out the value of the car at the end of 2021.

   $8460 \times 0.85^2 = 6112.35$

   £ 6112.35

   (Total 3 marks)

Contents ⌂

5. Aidan bought a piano for £3200.
   In each year the value of the piano increases by 11% of its value at the start of that year.

   a) Find the value of the piano after one year.

   3200×1.1

   £ ....3552.............
   (2)

   b) Calculate after how many complete years the value of the piano will be at least £4000.

   $3200 \times 1.11^3 = 4376.4192$

   ....3 years............
   (2)
   (Total 4 marks)

6. Charlie bought a new car. Each year, the value of her car depreciated by 7%.
   Calculate the number of years after which the value of her car was 65% of its value when new.

   $0.93^\wedge < 0.65$

   $0.93^6 = 0.6469...$

   ....6 years............
   (Total 3 marks)

7. Henry invests £3500 at a compound interest rate of 2% per annum.
   At the end of n complete years the investment has grown to £4100.81
   Find the value of n.

   $3500 \times 1.02^\wedge = 4100.81$

   ....8 years............
   (Total 3 marks)

8. £5600 is invested at 2.8% compound interest per annum.
   How many years will it take for the investment to exceed £7000.

   $5600 \times 1.028^9 = 7180.026...$

   ....9 years............
   (Total 3 marks)

9.   Dev buys a new washing machine.
      The value of the washing machine depreciates by 20% each year.

      a)   Dev says 'after 5 years the machine will have no value'.
            Dev is wrong. Explain why.

            The 20% will change in value each
            year, not stay the same.
                                                                              (1)

      Dev wants to work out the value of the washing machine after 2 years.

      b)   By what single decimal number should Dev multiply the value of the washing
            machine when new?

            $0.8^2 = 0.64$

                                                         0.64
                                                                              (2)
                                                              (Total 3 marks)

10.   Valda wants to invest £3000 for 2 years in the same bank.

The Money Box	The Best Bank
Compound Interest	Compound Interest
3% for the first year 1% for each extra year	4% for the first year 0.5% for each extra year

      At the end of 2 years, Valda wants to have as much money as possible.
      Which bank should she invest her £3000 in?

      Money Box   Best Bank

      3000 × 1.03 × 1.01   3000 × 1.04 × 1.005

      = £3120.90   = £3135.60

                          Best Bank

                                                              (Total 4 marks)

<u>Useful websites:</u>

www.piximaths.co.uk

www.mathswatchvle.com

www.corbettmaths.com

www.mymaths.co.uk

www.drfrost.com

www.bbc.co.uk/schools/gcsebitesize/maths

Remember: Do your best;
it is all you can do ☺